The Book Review Column[1]

by William Gasarch
Department of Computer Science
University of Maryland at College Park
College Park, MD, 20742
email: gasarch@cs.umd.edu

In this column we review the following books.

1. **Combinatorial Optimization: Packing and Covering** by Gérard Cornuéjols. Review by Brian Borchers. This book is about solving Packing and Covering problems using linear and integer programming.

2. **Ordered Sets: An Introduction** by Bernd S. W. Schröder. Review by Radim Belohlavek. This book is about the algebraic, combinatorial, and algorithmic aspects of ordered sets.

3. A Joint review of

   (a) **General Lattice Theory (Second Edition)** by George Grätzer.

   (b) **The congruences of a finite lattice: a proof-by picture approach** Also by George Grätzer.

   Reviews by Jonathan A. Cohen. The first book covers many of the aspects of Lattice Theory. The second one is more focused on work arising from a particular conjecture and its solution.

4. **Applied Combinatorics on Words** by M. Lothaire. Review by Maulik Dave. The book deals with algorithms, and theoretical properties of texts. Most of algorithms are expressed in pseudo code. They are accompanied by their proofs of correctness and examples.

5. **Computation engineering: applied automata theory and logic** by Ganesh Lalitha Gopalakrishnan. Review by S. C. Coutinho. This is a book on automota theory with some material on applications and logic.

Books I want Reviewed

If you want a FREE copy of one of these books in exchange for a review, then email me at gasarchcs.umd.edu

Reviews need to be in LaTeX, LaTeX2e, or Plaintext.

Books on Algorithms and Data Structures

1. *Visibility Algorithms in the Plane* by Ghosh.

2. *How to think about Algorithms* by Jeff Edmonds.

3. *Algorithms on Strings* by Crochemore, Hancart, and Lecroq.

4. *Algorithms for Statistical Signal Processing* by Proakis, Rader, Ling, Nikias, Moonen, Proudler.

5. *Nonlinear Integer Programming* by Li and Sun.

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6. *Binary Quadratic Forms: An Algorithmic Approach* by Buchmann and Vollmer.
7. *Curve and Surface Reconstruction: Algorithms with Mathematical Analysis* by Dey

Books on Cryptography, Security, and Coding Theory

1. *Concurrent Zero-Knowledge* by Alon Rosen.
2. *Cryptography and Computational Number Theory* edited by Lam, Shparlinski, Wang, Xing.
3. *Coding, Cryptography, and Combinatorics* edited by Feng, Niederreiter, Xing.
4. *Formal Correctness of Security Protcols* by Bella
5. *Coding for Data and Computer Communications* by David Salomon.
6. *Block Error-Correcting Codes: A Computational Primer* by Xambo-Descamps.

Combinatorics Books

1. *Computationally Oriented Matroids* by Bokowski
2. *A Course on the Web Graph* by Bonato
3. *Algorithmic Combinatorics on Partial Words* by Blanchet-Sadri.

Logic and Verification Books

1. *Software Abstractions: Logic, Language, and Analysis* by Jackson.
2. *Formal Models of Communicating Systems: Languages, Automata, and Monadic Second-Order Logic* by Benedikt Bollig.
3. *Modelling Distributed Systems* by Fokkink.

Misc Books

1. *Higher Arithmetic: An algorithmic introduction to Number Theory*
2. *A Conscise introduction to Data Compression* by Salomon.
3. *Putting Auction Theory to Work* by Paul Milgrom.
4. *Difference Equations: From Rabbits to Chaos* by Cull, Flahive, and Robson.
5. *Dissemination of Information in Optical Networks* By Bandyopadhyay.

Review [2] of
Combinatorial Optimization: Packing and Covering
Author of Book: by Gérard Cornuéjols
SIAM, 2001, 132 pages
Review written by Brian Borcher

This book is based on a series of ten lectures given by Cornuéjols at a CBMS–NSF regional conference in 1999. It is a monograph on the polyhedral combinatorics of packing and covering problems. The fundamental approach is to formulate a combinatorial optimization problem as an integer linear programming problem and then determine conditions under which the feasible region of the linear programming (LP) relaxation of the problem has integral vertices. In such cases, the problem can be solved efficiently by simply solving the LP relaxation.

For example, the weighted vertex cover problem can be formulated as an integer programming problem

$$\begin{array}{lccc} \min & wx & & \\ & Mx & \geq & 1 \\ & x & \geq & 0 \\ & x & \text{integer.} & \end{array}$$

Here, M is a 0–1 matrix with $M_{i,j} = 1$ when edge i is incident on vertex j. The vector w gives nonnegative vertex weights. In an optimal solution to this integer programming problem, the 0–1 vector x encodes an optimal weighted vertex cover.

Because the vertex cover problem is NP–Hard, this integer programming formulation is also NP–Hard. However, there are certain restricted versions of the problem that are efficiently solvable. For example, it can be shown using König's theorem that if the graph is bipartite, then the polyhedron

$$P = \{x | Mx \geq 1, x \geq 0\}$$

has integral vertices. In this case, we can efficiently solve the vertex cover problem by solving the LP relaxation of the integer programming formulation of the vertex cover problem.

A clutter is a combinatorial structure defined by a set V and a family $E(V)$ of subsets of V with the property that no subset in $E(V)$ is contained within another of the subsets. The corresponding clutter matrix M is a 0–1 matrix with $M_{e,v} = 1$ if $v \in e$. Given a clutter matrix M and weights w, we can consider the linear programming problem

$$\begin{array}{lccc} \min & wx & & \\ & Mx & \geq & 1 \\ & x & \geq & 0 \end{array}$$

and its dual

$$\begin{array}{lccc} \max & y1 & & \\ & yM & \leq & w \\ & y & \geq & 0. \end{array}$$

A clutter is said to pack if both LP's have optimal integral solutions when $w = 1$. The clutter is said to have the packing property if both LP's have optimal integral solutions for all weight vectors w with 0, 1, and infinite weights. Clearly, the packing property implies that a clutter packs. If

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both LP's have integral optimal solutions for all integral weight vectors, then the clutter is said to have the max–flow min–cut (MFMC) property. The MFMC property implies the packing property. It has been conjectured that a clutter has the MFMC property if and only if it has the packing property.

For example, consider a directed graph (N, A), with designated nodes s and t. Let $V = A$ and let $E(V)$ be the set of all s–t paths in the directed graph. The s–t max–flow problem with nonnegative integer arc capacities w can be formulated as

$$\begin{array}{rrcl} \max & y1 & & \\ & yM & \leq & w \\ & y & \geq & 0. \end{array}$$

By the Ford–Fulkerson algorithm, we know that this LP has at least one integral optimum solution for any vector of nonnegative integer weights w. The dual of this linear programming problem is

$$\begin{array}{rrcl} \min & wx & & \\ & Mx & \geq & 1 \\ & x & \geq & 0. \end{array}$$

By the max–flow min–cut theorem, we know that there is a 0–1 optimal solution to the dual LP corresponding to a minimum cut in the directed graph. Thus the clutter of s–t paths on a digraph has the MFMC property.

Packing problems can be analyzed in a similar manner. The weighted node packing problem can be formulated as

$$\begin{array}{rrcl} \max & wx & & \\ & Mx & \leq & 1 \\ & x & \geq & 0 \\ & x & \text{integer}. & \end{array}$$

The graph G and matrix M are said to be perfect if the polyhedron

$$Q = \{x | Mx \leq 1, x \geq 0\}$$

has integral vertices. Although the node packing problem is in general NP-Hard, when G is perfect we can efficiently solve the node packing problem by optimizing over Q.

Many families of perfect graphs are known. For example, a graph is called chordal if for every cycle of four or more vertices there is an edge that connects two nonadjacent vertices in the cycle. It can be shown that chordal graphs are perfect. There has also been considerable interest in classifying those graphs that are not perfect. An odd hole is a graph which consists of a chordless cycle with an odd number of vertices. Berge's famous perfect graph conjecture asserted that the only minimal imperfect graphs are odd holes and the complements of odd holes. This famous conjecture was finally proved shortly after the publication of this book[1].

Packing and covering properties of various clutters including two–commodity flows, r–cuts and r–arborescences, T–cuts and T–joins, and odd cycles are the subject of chapters one through five of the book. In chapters six and seven, the approach is extended to balanced and totally unimodular matrices with 0, 1, and -1 entries. In chapters eight, nine, and ten, Cornuéjols discusses decompositions of matroids, balanced matrices, and perfect graphs.

The book is written in very dense definition–theorem–proof style, with relatively little motivation. Although a number of exercises are scattered throughout the book, the style of presentation makes this book unsuitable for use as an introduction to the topic. Readers looking for an applications oriented introduction to this material will find an excellent one in the combinatorial optimization textbook by Nemhauser and Wolsey[2].

Cornuéjols book can be recommended as an authoritative reference for graduate students and researchers working in this area. However, it is already beginning to be outdated by new developments. The author offered a prize of $5,000 for each of eighteen conjectures. Including the proof of the strong perfect graph conjecture, five of these eighteen conjectures have now been solved. An updated edition would be welcome.

References

[1] M. Chudnovsky, N. Robertson, P.D. Seymour, and R. Thomas. The strong perfect graph theorem. *Annals of Mathematics*, 164:51–229, 2006.

[2] George L. Nemhauser and Laurence A. Wolsey. *Integer and Combinatorial Optimization*. Wiley-Interscience, 1988.

Review[3] of
Ordered Sets: An Introduction
Author of book: Bernd S. W. Schröder
Publisher: Birkhäuser, Boston, 2003
391 pages

Author of review: Radim Belohlavek
rbelohla@binghamton.edu

1 Introduction

The book under review deals with ordered sets. It provides a comprehensive account of the theory of ordered sets including the algebraic, combinatorial, and algorithmic aspects.

The area of ordered sets can be seen as lying in between graph theory and lattice theory, see [1, 4, 5]. An ordered set is a pair $(P, \leq)$ where P is a non-empty set (so-called support set) and $\leq$ is a partial order on P. That is, $\leq$ is a binary relation on P which is reflexive, antisymmetric and transitive. $\leq$ being a binary relation means that $\leq$ is a subset of the Cartesian product $P \times P$, i.e. $\leq$ contains (some) ordered pairs (p, q) of elements $p, q \in P$. Using infix notation, $p \leq q$ denotes that the pair (p, q) belongs to $\leq$, i.e. p and q are related. Reflexivity of $\leq$ means that for each $p \in P$: $p \leq p$; antisymmetry means that for each $p, q \in P$: ($p \leq q$ and $q \leq p$) implies $p = q$; transitivity of $\leq$ means that for each $p, q, r \in P$: ($p \leq q$ and $q \leq r$) implies $p \leq r$. Instead of "ordered set", one uses also "partially ordered set" or just "poset". Ordered sets (finite ones) can be visualized using diagrams (called Hasse diagrams or line diagrams). Diagrams are an important tool in ordered sets (one thinks of ordered sets in terms of their diagrams). A diagram of an ordered set $(P, \leq)$ is in fact a graph of relation $\leq - \leq^2$, drawn in a bottom-up direction. That is, we draw a line from p

[3] ©Radim Belohlavek 2008

to q heading up if and only if $p \leq q$, $p \neq q$, and there is no r distinct from p and q such that $p \leq r$ and $r \leq q$.

Therefore, ordered sets being particular binary relations means they are particular directed graphs. On the other hand, an important particular case of ordered sets is that of lattice-ordered sets which can be identified with so-called lattices. A lattice-ordered set is an ordered set $(P, \leq)$ such that for every two (and therefore, any finite number of) elements p and q there exist both the infimum $\inf(\{p, q\})$ and the supremum $\sup(\{p, q\})$ of p and q. An infimum $\inf(X)$ of a subset X of P is the greatest lower bound of X (provided it exists), i.e. we have: (1) for each $p \in X$: $\inf(X) \leq p$, (2) if there is $q \in P$ such that for each $p \in X$ we have $q \leq p$, then $q \leq \inf(X)$. Dually, a supremum $\sup(X)$ of a subset X of P is the least upper bound of X (provided it exists). The two interfaces (graphs and lattices) are significant for research in ordered sets because methods and questions from graphs and lattices have been applied to ordered sets.

Ordered sets are among the basic concepts of mathematics and appear virtually in any particular area of mathematics (numbers with the usual ordering form an ordered set, a collection of subsets of a given set with subsethood relation is an ordered set, positive integers with divisibility form an ordered set, functions with pointwise ordering form an ordered set, etc.). More generally, they appear in areas where one deals with hierarchies and ordering of entities such as knowledge discovery and data analysis (hierarchical clustering, formal concept analysis), conceptual systems (ontologies), or sociology and management (structure of an organization). In a sense, every mathematician and computer scientist is familiar, explicitly or implicitly, with basic concepts from ordered sets such as least/greatest element, minimal/maximal element, infimum/supremum, and others. Thinking clearly about these concepts is a good reason for including parts of the theory of ordered sets in curricula (algebra or discrete mathematics courses). There are, however, many mathematically non-trivial results on ordered sets which go far beyond the above-mentioned concepts which "everybody knows". The mathematical study of ordered sets goes back to 1930s (perhaps even earlier, see e.g. Schröder E.: Vorlesungen über die Algebra der Logik, 3 vols. B.G. Teubner, Leipzig, 1890-1905) but it was not until the beginning of 1970s when the research started the results of which form the state of the art of ordered sets as presented in this book. In most cases, motivations for studying ordered sets have been purely mathematical. That is, ordered sets are nice mathematical objects and one wants to learn more about their structure. Ordered sets lead to interesting problems including combinatorial and algorithmical ones. However, as is happens time to time with purely abstract mathematical concepts, ordered sets have found interesting applications. Two such applications are mentioned below.

The first one is Formal Concept Analysis (FCA, see [3]) which is a method of data analysis pioneered by Rudolf Wille. In the basic setting, the input data forms a table with rows and columns corresponding to objects and (yes-or-no) attributes and with table entries indicating whether a particular object has a particular attribute. In FCA, one tries to extract (1) particular clusters from data, which are called formal concepts, and (2) particular dependencies, which are called attribute dependencies. A formal concept is a particular pair (A, B) where A and B are sets of objects and attributes. For instance, A being the collection of all dogs and B being a collection of attributes such as "barks", "has tail", etc., might be en example of a formal concept (concept of a dog). The natural subconcept-superconcept relation makes the set of all formal concepts a partially ordered set which happens to be a (complete) lattice, so-called concept lattice. A concept lattice associated to data shows all natural concepts hidden in the data. The concept of partial order is central to FCA which has found applications in many areas including information retrieval, web search, data mining, classification, knowledge representation, or software engineering.

The second one is using partially ordered sets as models or terrorist cells, where partial order models the inner structure of the cell. The goal is to break a cell as cheaply as possible, i.e. kill some of its parts so that information cannot flow within the cell. Mathematically, such problems lead to combinatorial problems in ordered sets and the reader is referred to [2].

2 Coverage

The book consists of twelve chapters and two appendices, an 18 page list of references, and a 15 page list of key words. There are no prerequisites to the text except for a certain level of experience with reading mathematical texts and mathematical maturity. The text is an introductory one. It covers advanced topics as well and some parts might be considered difficult to get through. Nevertheless, the text is very clearly written. Every chapter contains a list of exercises and a list of open problems and remarks. The open problems and remarks are very useful, particularly for people who want to do research in ordered sets themselves and/or are not interested just in the presented results but also in the "surrounding story".

Chapter 1 introduces basic notions related to ordered sets: the concept of ordered set, diagrams, examples, morphisms, i.e. structure-preserving mappings between ordered sets, automorphisms and fixpoints, and the like. Chapter 2 deals with chains, antichains, and fences. A chain in $(P, \leq)$ is a subset M of P such that every two $p, q \in M$, either $p \leq q$ or $q \leq p$. M is an antichain if neither $p \leq q$ nor $q \leq p$ is the case for arbitrary $p, q \in M$. A fence is a zig-zag shaped subset of an ordered set. Chains are related to Zorn Lemma and well-ordered sets and these topics are explained here. Furthermore, Dilworth's lemma saying that every finite ordered set with k being the number of its largest antichain can be decomposed into k chains, is covered with related results. The last topic here is connectedness, i.e. the property of being able to go from any element in an ordered set to any other via edges in the corresponding diagram. Chapter 3 studies minimal elements, maximal elements, upper bounds, lower bounds, infima, suprema and related issues such as how much information about the whole ordered set is given by partial information expressed in terms of maximal/minimal elements. Chapter 4 takes a look at retractions which are idempotent order-preserving mappings $r : P \to P$. That is, we have $p \leq q$ implies $r(p) \leq (q)$ and $r(r(p)) = p$ for each $p \in P$. Retractions are connected to fixed point property of ordered sets and are used for exploring the structure of ordered sets several ways. Basically, retractions enable one to reduce problems regarding an ordered set to problems regarding smaller sets. Chapter 5 studies lattices (see above) which are perhaps the most studied ordered sets. Issues including fixed point property, completion (how to embed an ordered set into a lattice), and particular properties of lattices such as distributivity. Truncated lattices, i.e. ordered sets in which any two elements which have a lower/upper bound have their infimum/supremum, are the subject of Chapter 6. The concept of a dimension of ordered sets, i.e. the smallest number k of chains on P whose intersection gives the order $\leq$ on P, is studied in Chapter 7. Chapter 8 deals with interval orders which naturally appear in task scheduling problems. Interval orders result as orders where elements of P are tasks with a beginning and ending in time and $p \leq q$ means that task p ends before task q begins. Chapter 9 studies the concept of a lexicographic sum, a particular way to obtain new ordered sets from given ordered sets. Chapter 10 studies homomorphisms (i.e., order-preserving mappings) and direct products of ordered sets. Chapter 11 studies various forms of a question of how many (non-isomorphic, i.e. different) ordered sets there are on P which has k elements. Algorithmic aspects of ordered sets are presented in Chapter12, which is the longest chapter in the book (42 pages). The chapter includes preliminaries on algorithms and complexity (time complexity, NP and NP completeness) and includes selected problems for which their solvability and complexity is discussed. Focus is on problems related to whether an ordered set has fixed point property. Appendix A is on algebraic topology, Appendix B shows an application of ordered sets in analysis, in particular how order-theoretical tools can replace functional-analytic tools.

3 Summary

There are two groups of people who will primarily profit from the book. First, researchers in the field of ordered sets and lattices and related fields like algebra, graphs, and combinatorics. The book provides an excellent look at the field with numerous remarks including historical remarks and open problems. Second, students who are looking for a PhD topic. The author presents the field of ordered sets in an attractive way and the many open problems presented in the book are invaluable. Mathematicians and computer scientists could use the book as a reference book, too, although they will probably consider many topics too specific to be of interest. The book is clearly on theory of ordered sets, not on the applications of ordered sets. As such (being a book on theory), the book is a success, it presents an in depth and up to date carefully written coverage of ordered sets.

References

[1] Birkhoff G.: *Lattice Theory.* 3rd Ed. Providence, RI, Amer. Math. Soc., 1967.

[2] Farley J. D.: Breaking Al Qaeda Cells: A Mathematical Analysis of Counterterrorism Operations (A Guide for Risk Assessment and Decision Making). *Studues in Conflict & Terrorism* **26**(2003), 399-411.

[3] Ganter B., Wille R.: *Formal Concept Analysis. Mathematical Foundations.* Springer, 1999.

[4] Gratzer G. A.: *General Lattice Theory.* Birkhauser (1998, 2nd ed.).

[5] Priestly H. A., Davey B. A.: *Introduction to Lattices and Order.* Cambridge, England, Cambridge University Press, 1990.

Joint review[4] of
General Lattice Theory (Second Edition)
Author of Book: George Grätzer
Birkhauser, 688 pages
and
The congruences of a finite lattice: a proof-by picture approach
Author of Book: George Grätzer
Birkhauser, 282 pages

Author of Review: by Jonathan A. Cohen

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1 Introduction

The study of lattice theory as a mathematical subject of independent interest originally arose in Garrett Birkhoff's work in the 1930's as a generalisation of the theory of boolean algebras. As such, it has always enjoyed a close affinity with logic. Just as boolean algebras form algebraic models of classical logic, broader classes of lattices provide the foundation for algebraic models of various nonclassical logics. More recently, certain classes of lattices have become fundamental ingredients in the denotational semantics of functional programming languages.

The first book under review is an exposition of the general theory of lattices and is an update of the original edition published in 1978. The second edition makes no substantial alterations to the first, but adds eight appendices. The appendices are contributed by experts in the various fields and sketch some more recent developments in the area. The author focuses his attention on results in pure lattice theory, generally ignoring applications of lattice theory (with the notable exception of projective geometry). To a large extent the focus is adhered to by the authors of the appendices.

The second book under review investigates work arising from a famous conjecture in lattice theory. The author has played a fundamental rôle in the development of the theory and the book collects together and unifies many of the results that are scattered through the literature. A novel pedagogical aspect of the book is the notion of a "proof-by-picture", which presents a graphically oriented sketch of a proof before giving the formal proof itself. The book has a companion wesite at `http://www.maths.umanitoba.ca/homepages/gratzer/MathBooks/lectures.html`, which provides additional material relating to the various proofs-by-pictures that appear in the book.

2 Overview of "General lattice Theory"

The author uses the development of the theory of distributive lattices as a model for the development of lattice theory as a whole. This is emphasised by the central rôle that distributive lattices play in the book. Apart from making the unusual decision to introduce distributive lattices before modular lattices (the former being a subclass of the latter), distributive lattices recur repeatedly as congruence lattices of various other classes of lattices.

Chapter I: First Concepts. A lattice is a partially ordered set in which the greatest lower bound and least upper bound of any pair of elements always exists. Equivalently, one may view a lattice as a set equipped with two binary operations: $\wedge$ ("meet") and $\vee$ ("join") satisfying a certain finite set of identities. One typically views "$\wedge$" as representing the greatest lower bound and "$\vee$" as representing the least upper bound of two elements.

The first chapter introduces the reader to some of the basic constructions and notions of lattice theory and follows the same structure as the book as a whole. Amongst the important notions touched on are congruences, distributive lattices, modular lattices, complementation, boolean algebras and free lattices. A congruence is an equivalence relation on a lattice that respects the operations of meet and join. One of the important results of the chapter is that the collection of all congruences of a lattice, L, forms a lattice, $\mathrm{Con}(L)$ and that $\mathrm{Con}(L)$ is complete, meaning that the greatest lower bound and the least upper bound of any subset of $\mathrm{Con}(L)$ always exists.

Chapter 2: Distributive Lattices. A lattice is distributive if it satisfied the identity $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$. The chapter starts with the fundamental result that a lattice is distrubutive if and only if it is isomorphic to a ring of sets. It then goes on to discuss free distributive lattices and free boolean lattices. Following this, it is shown that, given any lattice L, the lattice of congruences

Con(L) is distributive and algebraic (by a theorem of Birkhoff and Frink, a lattice is algebraic if and only if it is isomorphic to the lattice of subalgebras of some universal algebra). Another major topic of the chapter is Stone's representation theorem, which provides a topological interpretation of boolean algebras. Also covered is the topic of pseudocomplementation and Birkhoff's subdirect representation theorem is proved.

Chapter 3: Congruences and Ideals. This chapter is a collection of results on various types of ideals and congruences of a general lattice that to some extent mimic the behaviour of those for distributive lattices.

Chapter 4: Modular and Semimodular Lattices. A lattice is modular if every pair of elements $x \geq z$ satisfies $x \wedge (y \vee z) = (x \wedge y) \vee z$. A lattice is semimodular if, for any pair of elements $a \prec b$ and any other element c, either $a \vee c \prec b \vee c$ or $a \vee c = b \vee c$. The main focus of this chapter is a certain class of semimodular lattices known as "geometric lattices". Taking a geometry to be a set equipped with a distinguished collection of subsets called subspaces, satisfying some closure properties, the chapter goes on to show that every geometric lattice is isomorphic to the lattice of subspaces of some geometry.

The collection of all partitions of a set forms a lattice when ordered by refinement. The chapter shows that every such partition lattice is a simple geometric lattice. Partition lattices play the same rôle in lattice theory as permutation groups play in group theory. The fundamental result that every lattice embeds into a partition lattice is proved, as well as the famous corollary of Whitman that every lattice can be embedded in the lattice of subgroups of some group.

Following on from this, the chapter provides a study of projective geometry from a lattice-theoretic perspective, with a particular focus on Desarguesian projective geometries.

Chapter 5: Varieties of Lattices. While the previous chapters studied intrinsic properties of lattices, this chapter studies general universal algebraic properties of classes of lattices. Birkhoff's variety theorem is proved, which states that models of equational theories are equivalent to homomorphic images of subalgebras of products of algebras. Several variants utilising subdirect products and ultraproducts are also presented. A brief investigation of the lattice of varieties of lattices is conducted, wherein it is shown that this lattice is distributive and dually algebraic. The chapter also proves that the variety generated by a finite lattice has a finite equational basis and examines various amalgamation properties.

Chapter 6: Free Products. The free product of two lattices is the lattice freely generated by their union. This chapter investigates various structures arising from free products and proves that a lattice is free if and only if it is the free product of a family of one element lattices. Applications of the concepts are discussed, including the results that every lattice can be embedded into a uniquely complemented lattice and that every monoid can be represented as the bound-preserving endomorphism semigroup of a bounded lattice.

Appendix A. Retrospective (G. Grätzer). This appendix collects together several major advances since the first edition, as well as solutions to various open problems stated in the first edition.

Appendix B. Distributive Lattices and Duality (B.A. Davey and H.A. Priestley). This appendix briefly discusses Priestley's duality theorem for distributive lattices, which generalises Stone's theorem from Chapter 2. Also discussed is the duality theory of distributive lattices with additional operators, which arise in the model theory of modal logics.

Appendix C. Congruence Lattices (G. Grätzer and E. T. Schmidt). This appendix discusses some representation theorems of finite distributive lattices as congruence lattices of various types of lattices. A very brief discussion of the infinite case is also provided.

Appendix D. Continuous Geometry (F. Wehrung). This appendix provides an overview of some results on continuous geometries, which are continuous, complemented modular lattices. The appendix explores the relation between such lattices and von Neumann regular rings (the lattice of right ideals of such a ring is complemented and modular).

Appendix E. Projective Lattice Geometries (M. Grefarath and S. E. Schmidt). This appendix continues the lattice-theoretic investigation of Desarguesian projective geometry initiated in Chapter 4.

Appendix F. Varieties of Lattices (P. Jipsen and H. Rose). This appendix is a deeper investigation of the topics introduced in Chapter 5. Special emphasis is placed on rôle that the sublattice of varieties of distributive lattices plays in the lattice of varieties of lattices.

Appendix G. Free Lattices (R. Freese). This appendix collects together some results on free lattices and includes $O(n^2)$ algorithms for testing various properties of a finite lattice with n elements.

Appendix H. Applied Lattice Theory: Formal Concept Analysis (B. Ganter and R. Wille). This appendix surveys the field of formal concept analysis, which is a lattice-theoretic approach to data management and mining.

3 Overview of "The congruences of a finite lattice"

The congruence lattice of a finite lattice is always a distributive lattice. A theorem of Dilworth dating from 1944 obtains the converse: every finite distributive lattice is isomorphic to the congruence lattice of some finite lattice. This theorem is the starting point of the book, which seeks representation theorems of finite distributive lattices as congruence lattices of lattices with additional properties.

One might expect that in the general case, every distributive algebraic lattice is isomorphic to the congruence lattice of some lattice. For over sixty years, this was one of the major conjectures of lattice theory and a driving motivation for much research. However, subsequent to the publication of the book, F. Wehrung surprised many researchers by showing that the conjecture is false. His example of a distributive algebraic lattice that is not isomorphic to the congruence lattice of any lattice has $\aleph_{\omega+1}$ compact elements and appears in [Weh07]. Since Wehrung's result appeared after the present book went to press, no mention of it is made. However, since the book focuses on the finite case, it retains its interest.

The book is separated into five main parts, each consisting of several chapters.

Part I: A brief introduction to lattices. The three chapters that make up this part give a very fast introduction to the concepts from general lattice theory that are required in the book. Several results are presented without proof. The material includes basic results on lattices, direct and semidirect products, modular and distributive lattices and congruences. The fundamental notion of a congruence preserving extension is provided. For lattices $K \leq L$, we say that L is a congruence-preserving extension of K if for every congruence $\Theta \in \mathrm{Con}(K)$, there is a unique congruence $\Psi \in \mathrm{Con}(L)$ such that $\Psi|_K = \Theta$.

Part II: Basic Techniques. The three chapters that make up this part describe three different techniques for constructing congruence-preserving extensions.

The first of these techniques deals with chopped lattices: algebraic structures akin to lattices, except where the operation of join is only partially defined. It is shown that the lattice of ideals of a chopped lattice K is a congruence-preserving extension of K.

The second technique deals with special elements known as "boolean triples". For a lattice L, the collection of boolean triples of L, denoted $M_3[L]$, is a subset of L^3. Moreover, if L has a bottom element 0 and $\varphi : L \to M_3[L]$ is the map that sends $x \mapsto \langle x, 0, 0 \rangle$, then $M_3[L]$ is a congruence-preserving extension of $\varphi(L)$.

The third techniques involves the notion of a cubic extension. This is a more technically involved construction and produces a congruence-reflecting extension of a finite lattice, which is a somewhat weaker property than congruence preservation. The construction works as follows. For a lattice K, let $\mathrm{Con}_M(K)$ denote the congruences on K that are meet-irreducible. For $\Theta \in \mathrm{Con}_M(K)$, we can extend the quotient lattice K/Θ to a finite bounded simple lattice. $\mathrm{Cube}(K)$ is the product of all the simple lattices arising in this manner. Then, $\mathrm{Cube}(K)$ is a congruence-reflecting extension of K.

Part III: Representation Theorems. The five chapters of this part use the techniques of Part II, in particular chopped lattices, in order to obtain various flavours of representation theorems for distributive lattices.

The starting point is Dilworth's theorem, for which a proof utilising chopped lattices is presented. The following chapter gives a combinatorial version of Dilworth's theorem. In particular, it shows that a finite distributive lattice having $n \geq 1$ join-irreducible elements is isomorphic to the congruence lattice of a planar lattice with $O(n^2)$ elements. Moreover, this bound is shown to be best possible. Focus then shifts to representation by semimodular lattices and it is shown that every finite distributive lattice can be represented as the congruence lattice of a planar semimodular lattice having $O(n^3)$. It is not known whether this bound is best possible.

The final two chapters show that every finite distributive lattice can be represented as the congruence lattice of a modular lattice and of a uniform lattice, respectively. A lattice L is uniform if, for any $\Theta \in \mathrm{Con}(L)$, any two congruence classes of Θ have the same size.

Part IV: Extensions. The five chapters that make up this part build on the representation theorems together with the technique of cubic lattices in order to provide congruence-preserving extensions of finite lattices having specified properties. The first three chapters show that every finite lattice has a congruence-preserving extension to a finite lattice that is sectionally complemented, semimodular, or isoform; respectively. The following chapter shows that given any finite lattice K and finite group G, there is a congruence-preserving extension of K whose automorphism group is isomorphic to G. The final chapter of this part discusses a certain congruence "destroying" extension technique that applies mainly in the infinite case.

Part V: Two lattices.. The three chapters that make up this part explore the way in which the congruence lattices of two finite distributive lattices can be related. The first chapter explores what happens for a sublattice $K \leq L$ and the subsequent chapter explores the relation betweeen $\mathrm{Con}(K)$ and $\mathrm{Con}(L)$, where K is an ideal of L. The last chapter defines and explores a certain notion of tensor product. Given a finite lattice L and a finite distributive lattice D, we define $L[D]$ to be the lattice of all order-preserving maps from the join-irreducible elements of D to L. The main purpose of the chapter is to prove that $\mathrm{Con}(A[B])$ is isomorphic to $(\mathrm{Con}(A))[\mathrm{Con}(B)]$, where A and B are any nontrivial finite lattices.

4 Opinion

General Lattice Theory. The first edition of this book rapidly implanted itself as the standard introductory and reference work in lattice theory. As a reference work for lattice theory, the second edition still excels.

The fragmented nature of the second edition, together with the parochial choice of topics makes it less suitable as an introductory text. An example of the fragmentation brought upon by the appendices is the treatment of the important concept of Priestley duality for distributive lattices which, while covered in Appendix B, is not mentioned in Chapter 2.

Despite these failings, the book remains a wonderful resource for lattice theory and can be fruitfully used as an additional text in lattice theory courses at the advanced undergraduate or beginning graduate level.

The Congruences of a Finite Lattice. This is a much more tightly focused book. While it is essentially self-contained, it is complemented very well by General Lattice Theory. It covers very many interesting results on congruence lattices of finite distributive lattices and includes a wealth of open problems. The author's decision to provide a graphical sketch of proofs before formal proofs works very well. The book is very well suited for self-study as an entrée to this research area for readers already acquainted with the basics of lattice theory.

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A Brief response to J. A. Cohen's joint review[5]

Response by G. Grätzer
Department of Mathematics
University of Manitoba
Winnipeg, MB R3T 2N2, Canada
gratzer@me.com
`http://server.math.umanitoba.ca/homepages/gratzer/`

would not call this a rebuttal. I agree with most everything the reviewer writes, and I thank him for his effort. Nevertheless, as the author of the two books under review, I see some things differently.

- From the review:

  "The author focuses his attention on resultsin pure lattice theory, generally ignoring applications of lattice theory (with the notable exceptionof projective geometry)."

  Quite so. Birkhoff's *Lattice Theory* (all three editions) was all about finding examples of lattices to justify the theory. I took it for granted that lattice theory was interesting.

- From the review:

  "The author uses the development of the theory of distributive lattices as a model for the development of lattice theory as a whole. This is emphasised by the central role that distributive

[5]The research of the authors was supported by the NSERC of Canada.

lattices play in the book. Apart from making the unusual decision to introduce distributive lattices before modular lattices (the former being a subclass of the latter), distributive lattices recur repeatedly as congruence lattices of various other classes of lattices."

In most lattice theory books preceding mine, distributive lattices came late. I put them before all the others (semimodular, modular, etc.) because I wanted to get to interesting results fast. It is also true that a lot of what we learn from distributive lattices apply to other classes, think of distributive and standard elements and ideals. But basically, I just wanted to get the readers' attention before they loose interest.

- I think the referee's view of the second edition of GLT is somewhat different from mine. I do not believe that the second edition is fragmented. Rather, the second edition is the unchanged first edition with eight survey articles as appendices.

  When I wrote the first edition, in 1968–1977, I "knew" lattice theory. We spent a lot of time in my seminar discussing papers that were not included in the book—we deemed them not important enough. The second edition was completed by the end of 1998. Between 1977 and 1998, 7,200 papers and books were published with lattice theory as the primary classification (about 12,250 with lattice theory as the primary or the secondary classification). And there were only 7,665 days!

  So I came to the conclusion that there is no way I can pretend to know the field any longer. The first edition is a good introduction to the field, and the developments of the 20 some years in between the two editions should be surveyed by experts.

- From the review:

  "An example of the fragmentation brought upon by the appendices is the treatment of the important concept of Priestley duality for distributive lattices which, while covered in Appendix B, is not mentioned in Chapter 2."

  Do remember that the Priestley duality for distributive lattices established itself in the early to mid 1980-s. Almost a decade too late for the first edition.

- From the review:

  "The first edition of this book rapidly implanted itself as the standardintroductory and reference work in lattice theory."

  We can actually quantify this. E. Garfield [1] points out that this book put me on the list of the 200 pure mathematicians most cited in 1978 and 1979.

- From the review:

  "Since Wehrungs result appeared after the present book went to press, no mention of it is made. However, since the book focuses on the finite case, it retains its interest."

  I think I do explain in the Introduction of the congruence lattice book that this field splits into two topics: the finite case and the infinite case, with no overlap. Wehrungs amazing result was proved after the book was published. Had it been proved earlier, there would have been one more reference in the bibliography...

  The interested reader can read this story in my article [2].

References

[1] E. Garfield, *The 200 Pure Mathematicians Most Cited in 1978 and 1979,* Essays of an Information Scientist **5** (1982), 666–675.

[2] G. Grätzer, *Two problems that shaped a century of lattice theory.* Notices Amer. Math. Soc. **54** (2007), 696–707.

Review of
Applied Combinatorics on Words. [6]
by M. Lothaire
Published in 2005 by Cambridge University Press, 610 pages

Reviewer: Maulik Dave

1 Overview

Text manipulations play an important part in various kinds of computations. The subject of text manipulations, is a well developed subject in computer science. The book deals with algorithms, and theoretical properties of texts. Most of algorithms are expressed in pseudo code. They are accompanied by their proofs of correctness, and complexity analysis. Some of them are explained with discussions containing examples, and diagrams. The stress is on combinatorial properties. The book is a part of a series on encyclopedia of mathematics.

2 Content Summary

The book consists of 10 chapters.

2.1 Chapter 1 on basic algorithms.

The chapter begins by introducing commonly used terms on words, such as ordering, Hamming distances, factors, prefixes, and suffixes. A word w is called a factor of a word u if there exist words x, y such that $u = xwy$. The elementary algorithms related to factors are briefly presented. Hamming distance between two words represent the number of mismatches between them. Automata are explained in details. Determinization, and minimization algorithms on automata are described. Transuders are described with minimization algorithm. Discussion on parsing consists of both top down, and bottom up parsing. The chapter moves to statistics on words after parsing. The enumeration on words is introduced briefly. Probability distributions on words are described in details. The description includes entropies, and ergodic sources.

2.2 Chapter 2 on Indexes.

An automaton accepting suffixes of a word is called a suffix trie. Minimal such automaton is called suffix automaton. Construction algorithms for suffix trie, and automaton are described. Second

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part of the chapter concentrates on operations on a fixed text. Algorithms of suffix automata are applied. Algorithms of various operations on text are described. Operations include occurrences of factors, repetitions, search for forbidden words, patterns, and conjugates.

2.3 Chapter 3, and 4 on Natural Language Processing.

Basics of natural language processing are dealt. After a brief introduction to encoding, and tokenization, the morphological analysis is discussed in details. Applications of weighted automata, pattern matching, and recursive transition networks, are explained. An algorithm for composition of weighted transducers, is described. Algorithms for determinization, and minimization of weighted automata, are described. In the end, a discussion on speech recognition includes pronunciation, and acoustic models.

2.4 Chapter 5 on Network Expressions.

Inference of network expressions is explained in fifth chapter. Algorithms for star model, and clique model, are described. Algorithms for inference in network expressions with and without spacers (constrained and unconstrained) are described. The chapter ends with a section on open problems in network expressions.

2.5 Chapter 6 on Word Sequences.

The chapter is on statical aspects of word sequences. It centers around applications to biological sequences. Marcovian models are discussed in details. Theories of various distributions such as distribution of distance between word occurrences, and word count distributions are explained. How these theories can be applied to DNA sequences, is explained. The chapter ends by presenting some useful theorems in statistics.

2.6 Chapter 7 on Pattern Matching.

An analytical approach to pattern matching problems are discussed, in details. The stress is on probabilistic aspects. The problems discussed include exact string matching, generalized string matching, subsequence pattern matching, generalized pattern matching, and self repetitive pattern matching.

2.7 Chapter 8 on Periodicity.

Algorithms for various types of problems related to periodicity are explained. After defining squares, longest basic functions, and s factorizations are described as basic algorithmic tools. The problems addressed, include finding all maximal repetitions, finding repeats with a fixed gap, computing local periods, and finding approximate repetitions.

2.8 Chapters on Mathematical Structures.

The last two chapters deal with problems in mathematical structures, which involve words. Walks in sectors of a plane, forms words. Enumeration formulas of various types of walks are presented. Polyominoes, planes, morphic sequences, and automatic sequences are described in details.

3 Conclusion, and Comments

The book can be used as a textbook for an advanced course at graduate level. It is a good reference book to keep it. The first five chapters can be used a textbook reference for an elementary course on text manipulations. The list of references at the end of the book spans 24 pages. At the end of each chapter, notes on history is provided. Algorithms are described by tools such as pseudo code, diagrams, and examples. They are backed by detailed theoretical analysis.

Review[7]

Computation engineering: applied automata theory and logic

Author of Book: Ganesh Lalitha Gopalakrishnan

Springer (2006), 471 pages

Author of Review: S. C. Coutinho

collier@impa.br

1 Introduction

For well over a hundred years engineers have learned mathematics in order to be able to build bridges, houses and electrical circuits. For most of this time, this *engineering mathematics* consisted mostly of differential and integral calculus. However, as computers become more and more complex a new set of mathematical tools is proving to be as necessary a part of the *engineering mathematics* of those who work with them, as calculus has been for so many years. One of these new tools is the theory of automata.

One might think of automata theory as a set of mathematical models of several kinds of "disembodied" computers. These models describe the essence of what it means to do a computation, without any of the trappings related to the physical realization of such a machine. Although, this has traditionally been the point most often emphazised in courses on formal languages and automata, it is far from exhausting the importance of this subject.

Indeed, returning to the engineering point of view, which was our starting point, one may also argue that automata are now more important than ever as a means of certifying both hardware and software. For, as computers take care of a growing number of tasks, they also become more complex, and their hardware and software more prone to error. Haven't we all experienced this with our own personal computers? As the author says (p. 7), the "mantra" now seems to be: "reset, and move on."

Although we all feel annoyed when our computers crash this can hardly be called a catastrophe. But computers are also in control of more serious matters: air traffic, rockets and even machines used in radiation therapy. Take the last example. In 1987, a radiation therapy machine called Therac-25 caused the death of several cancer patients. The problem, it turned out, was overconfidence on the part of those who had redesigned its software: it was considered so reliable that the usual safety mechanisms were not used. Tragically, the software was faulty and, with no safety mechanism in place, some patients received a massive dose of radiation, which caused their deaths.

In order to avoid the risk of tragedies like this, companies are turning to the use of *formal methods* to help write precise and unambiguous specifications for both hardware and software. One is reminded of a similar trend that, in the 1950s, led to the formal specification of programming

[7]©S. C. Coutinho 2008

languages. Then, as now, the reason was the need for greater precision and fewer errors. Moreover, on both cases, the theory of formal languages and automata play a central rôle. Indeed, as the book under review shows, logic and automata theory can be combined to provide powerful tools for checking computer code; which is one of the reasons why they should also be in the toolkit of all of those who develop both hardware and software for today's computers.

2 Contents of the book

The book opens with a chapter called *Introduction* where the author describes the book's contents and justifies the importance of formal proof methods and the rôle of automata theory in computer engineering.

The remainder of the book can be roughly divided into three parts. The first, which goes from chapters 2 to 6, covers a number of mathematical and logic preliminaries. These include cardinality and the diagonalization process (chapter 3), binary relations (chapter 4), logic and proofs (chapter 5) and recursion (chapter 6). The second part is a standard course on formal languages and automata. Strings and languages are introduced in chapter 7, while finite automata are dealt with in chapters 8, 9 and 10. In chapter 11 we take a break to explore some connections with logic, via binary decision diagrams. This is followed by a chapter on the pumping lemma for regular languages, after which come context-free languages (chapter 13) and pushdown automata (chapter 14). At the end of this part we have a discussion of Turing machines (chapter 15) and their application to undecidability proofs (chapters 16 and 17) and complexity theory (chapter 19). Chapter 18 brings us back to logic, with a discussion of first order logic. In the final part we turn to applications, such as Pressburger arithmetic (chapter 20) and model checking (chapters 21, 22 and 23).

3 Opinion

There is no doubt that the book was written with great care and that it caters for a real need. As the author explains so eloquently in the introduction, formal methods of proof will be in ever greater demand as software and hardware become more and more complex. Also on the plus side, the book is written in a very lively style, which makes reading it quite pleasurable. Two features of the book deserve to be emphasized. The first is the effort the author has made to explain the motivation behind the more formal concepts and results. As part of that, interactive tools are used as a means to illustrate key concepts. The second are the many nice applications of automata that are peppered throughout the book. Since I was teaching formal languages and automata to undergraduates while I read the book, I had a chance to use many of these examples as illustrations in my course, with great success. Among the more elementary ones, my favourites are the applications of finite automata to error correcting codes (p. 176) and to binary decision diagrams (pp. 185ff).

However, I am afraid that, as the saying goes, too much of a good thing can be a bad thing. Indeed, it seems to me that one of the problems with the book is that it too often does not go beyond the heuristics of the main algorithms and results of automata theory. I think that this holds true even after one takes into account that this is *computer engineering*, rather than *computer science*; in other words, the emphasis is on applications, rather than fundamental theory. Although I agree with the author that “today’s students prefer learning *theory* as a tool rather than *theory for theory’s sake*” (p. xxvii), it is also true (at least in my experience) that they feel uncertain whether they have really understood a subject unless one provides some firm theoretical ground on which they can stand. For instance, to restrict the description of a rather complicated algorithm to one example of its applications would leave many of my students suffering from acute discomfort; see, for example, section 14.4 of chapter 14 (pp. 257ff).

To be honest, I felt myself very much in this position when I read the chapters on model theory, a subject about which I knew next to nothing. Although the introduction in chapter 21 is excellent, the treatment of temporal logics in chapter 22 is so rushed that I began to feel uncertain whether I was really getting the point or not. Having said that, it is likely that this book may be very nice as a textbook in the hands of an experienced lecturer. However, as far as self-study is concerned, it leaves much to be desired.

Finally, though this is no fault of the author, the book suffers from lack of proper proofreading. Besides the usual misspellings, there are sentences where whole words are missing and pages where the spacing suddenly changes for no seemingly good reason. The truth is that in these days of computer typesetting a professional proofreader is as necessary as ever–at least until we have formal methods capable of proofreading a typescript.

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