The Effects of Cabri Geometry for Exploring Geometry in Classroom

Tuyosi Nomura

t-nomura@ma3.justnet.ne.jp
Board of Education in Yuki City Japan

Abstract

In order to develop students' logical and mathematical reasoning, geometrical proof has been part of the junior high schools' mathematics curriculum in Japan; but recent research reported that only about twenty percent (20%) of students are able to solve geometrical proof-problems. The fact that many students could not succeed in writing a proof does not mean that they could not develop logical and mathematical reasoning. For developing their reasoning, the exploration of constructions must be taught a different way. Dynamic Geometry Software, like Cabri Geometry (Cabri) offers a new environment for exploring constructions. This paper discusses the change of reasoning that occurs using Cabri for the learning of geometry. Junior high school students, from twelve to fifteen years old, studied geometry using Cabri. Their change of reasoning is discussed from the following points: exploring the basic constructions in a different way than that of the text book, which could not be observed in the classroom before Cabri; and exploring and understanding the proposition before the proof. It was believed that proof was the only way to understand a proposition in the geometry classroom. But Cabri offers a different way of exploring figures, and also helps students' reasoning from the proposition to the conclusion. Through three years in-class observation, this study points out three major effects of using Cabri: visualizing the character of a figure clearly, a better understanding of the meaning of the theorem, and making clear what the students should be proving.

1 Introduction

A recent report started that Japanese students have strong knowledge and calculation skills. But there are many students who are not able to understand why and how to solve problems. They are good at memorizing given information, but they don't try to observe things personally and can't explain what they have learned. Recently, international research concluded that Japanese students' marks are high, but they don't have the desire to learn actively .

The most important thing about mathematics education is to encourage creativity and mathematical reasoning. Students need to learn geometry, especially the solving of geometrical proof-problems, in order to improve their logico-mathematical reasoning. Exploring figures with Cabri is effective in encouraging creativity

However, many mathematics teachers still use the pre-drawn figures for teaching proof and only teach one way of construction which is shown on the textbook.

Recently, there has been an increase in the number of teachers who use computer software, such as Cabri, in order to draw figures in their math classes. However, most teachers still only use pre-drawn figures. They don't prepare situations in which students draw figures themselves to solve geometric problems. There are few cases where teachers allow time for drawing figures in class.

Therefore, I prepared situations where Cabri should be used in the mathematics classroom to show how students could develop their reasoning by themselves using Cabri.

2 Object of Study

The aim of teaching mathematics in junior high school is to develop mathematical reasoning. It is believed that solving geometrical proof-problems contributes to improving students reasoning. But many students could not solve geometric proof problems. This paper focused on the exploration of constructions and moving figures for enhancing their reasoning. Dynamic Geometry Software like Cabri offers a new environment for exploring construction. This paper shows the change of students' reasoning that occurs from using Cabri for the learning of geometry.

In this paper, their change of reasoning is discussed from the following two points: First, how do students explore the basic construction using Cabri? Is it the same way of the text book? Second, how could students explore and understand the proposition using Cabri? Is it the same as not using Cabri?

3 Methods

Junior high school students, from 7th grade to 9th grade(twelve to fifteen years old), were observed studying geometry using Cabri. Students were observed over a three year. Students used Cabri in the classroom 10 hours per year. When students used Cabri, they had enough time to draw figures. The observations were analyzed using qualitative data: a work sheet including students reasoning and comments; the figures which students drew on the computer, which were used to analyze the procedures students used for drawing; and interviews with students during class.

4 Results of the Construction of Basic Figures with Cabri

Students had already learned the fundamentals of figures before the 7th grade. I allowed several hours in which students were given the opportunity to practice Cabri.

The following shows some of the examples of the basic construction that students have drawn using Cabri in the classroom. I was surprised at students' ways of construction. The following example shows that students can construct figures by themselves and improve their reasoning ability.

4.1 Example of students' construction

4.1.1 Construction of an isosceles triangle

For the construction of an isosceles triangle, most students tended to use a perpendicular bisector like Figure1-(1), which shown in the textbook.But some students drew their figure using the radius of a circle, as seen in Figure1-(2) which was not written in the textbook.

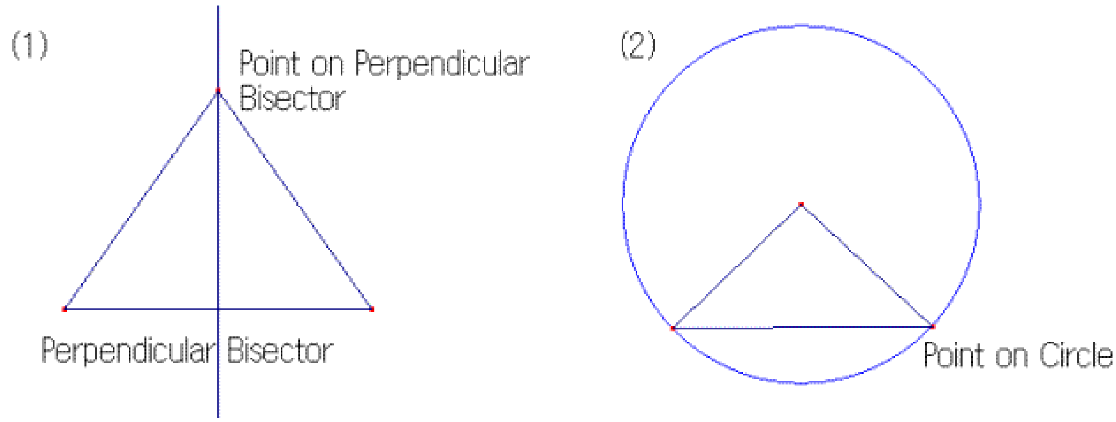

Figure1:Isosceles triangle

4.1.2 Construction of an equilateral triangle

I thought that all students would draw figures similar to that shown in Figure2-(1). I didn't expect the construction shown in Figure2-(2). But many students drew a figure similar to Figure2-(2) which applied the results of the construction of an isosceles triangle which they learned from Firue1-(1) & (2). It is also an evidence example where students construct figures by themselves.

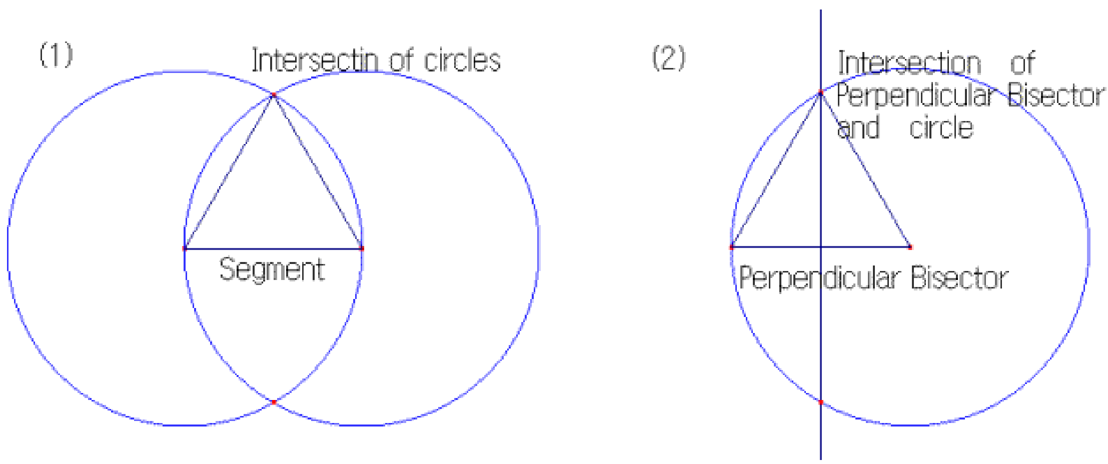

Figure2: Equilateral triangles

4.1.3 Construction of parallelograms

From the definition, most students drew a parallelogram like Figure3-(1). They could also draw a parallelogram by utilizing diagonal lines such as that shown in Figure3-(2).

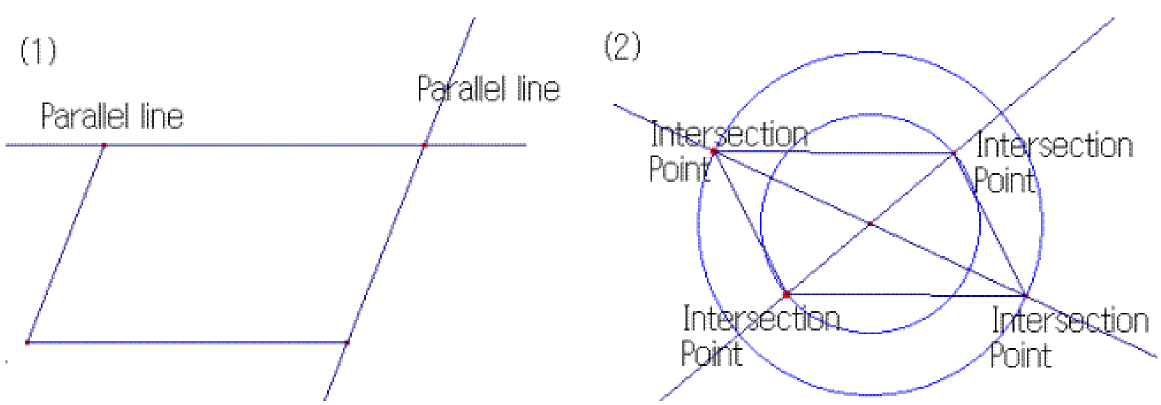

Figure3: Parallelograms

4.1.4 Construction of rhombuses

As for a rhombus, there were few students who drew the rhombus using three circles, as in Figure 4-(1). Many students used a perpendicular line as in the case of parallelogram in Figure 3-(2), and some of them used reflection, as in Figure4-(2).

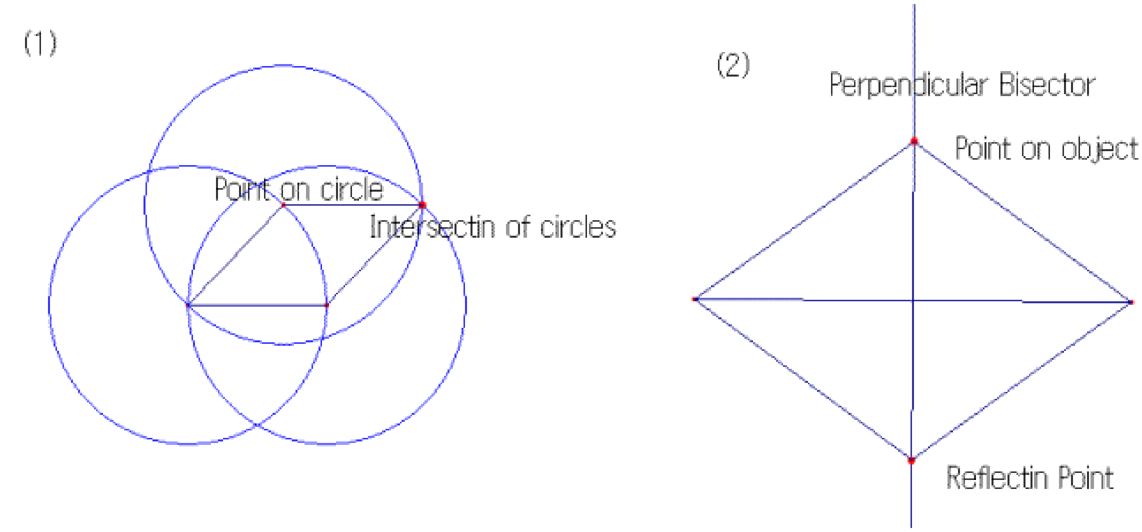

Figure4: Rhombuses

4.1.5 Construction of rectangles and squares

Figure5-(1) and Figure6-(1) were expected, based on the definitions in the textbook. Some students drew Figure5-(2) or Figure6-(2), which are not based on the definition. Therefore, these drawing are not easy, but the students were able to use the ideas shown in Figure5-(2) and 6-(2),and apply them to rectangles and squares. Students' own exploration of constructions were based on their own reasoning which we would not expect to see if the students used the textbook, a compass and a ruler. And in the case of the squares, students found many ways of drawing based on what they previously using Cabri.

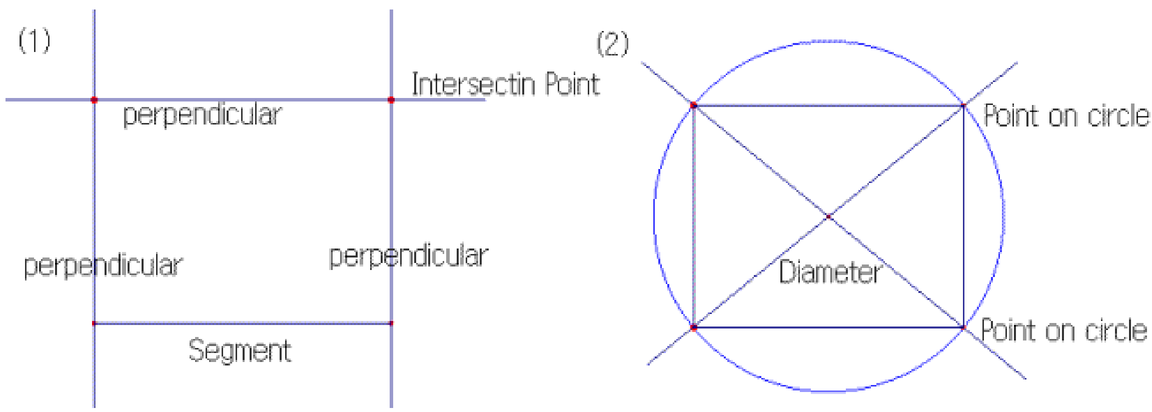

Figure5:Rectangles

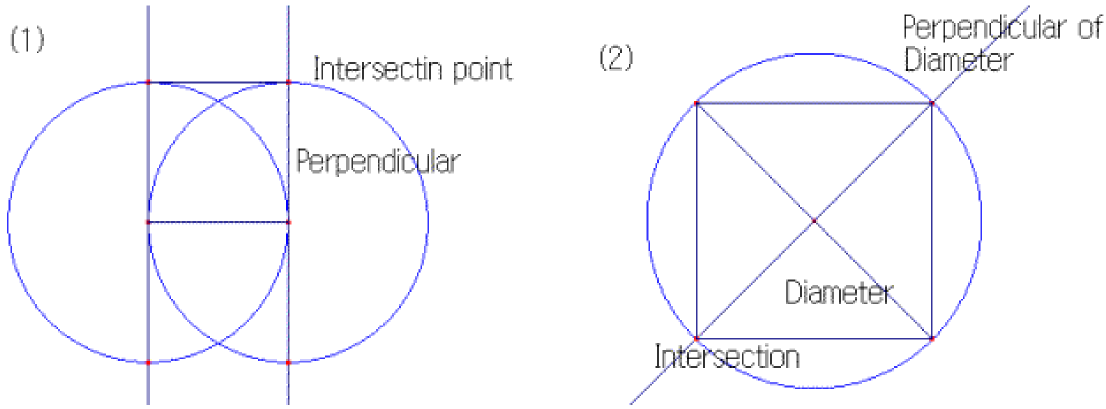

Figure 6: Squares

4.2 Impression of students who experienced reasoning by construction

Through lessons, students explored constructions by themselves using their own knowledge. When I asked students to explain their construction, they usually explained it using what they had learned previously in the classroom. Thus their explanations usually included their logical reasoning based on ways of construction shared in the classroom.Before Cabri, teachers did not give students time for exploring construction by themselves.

After the lessons in which they drew basic figures, students expressed the following. The following impressions are evidence that Cabri has changed the study environment for exploration, and enhanced their motivation for reasoning.

"I enjoyed making figures in various ways. There were many ways to make just one type of figure."

"I had trouble making the figures, but finally I was able to produce to them. I was happy with my achievement."

"I had lots of trouble and overcame the problems. I completed a rhombus with expansion reduction. It was very good and I was able to complete all the figures even though it was challenging. It was enjoyable."

"When I completed the figures I was very glad. The variety of methods was interesting and I was happy when I finished each figure. It was so enjoyable I want to do it again. I even want to color the figures."

"It was difficult but I understood the figures very well after I had drawn them."

"Making figures is similar to assembling a puzzle. After I had finished drawing them, I felt like an examination was over."

"It was easy to make a mistake because I had to use many kinds of points. I have not remembered all the various points yet, such as point, point of intersection and point on object."

"I was relieved when I finished a figure."

"I could make figures easily. When I drew figures myself. I could understand their properties. I was glad to have lessons in the computer room where I could use the figures."

These impressions show that we can give students a better environment for their exploration of construction by Cabri, rather than with compass, ruler and textbook.

5 Results of Exploration of Geometry and Proof

The study shows two examples. The first is an example of exploration and understanding of the proposition using Cabri. The second is an example of a proof using Cabri. In both cases, results are compared to results without using Cabri.

5.1 Explanation of theorem

The following is an example of an explanation of a theorem in the 9th grade text book. The theorem explains the following relationship between a circle chord and perpendicular bisector.

1. The perpendicular line bisects the chord.
2. The perpendicular bisector of the chord passes through the center of the circle.

When teachers taught the theorem only by explanation using chalk board, there were a lot of students who didn't understand these simple concepts. We could help them to understand more clearly what is an assumption and a conclusion through drawing figures. The following is a conversation between a student and a teacher about theorem 2 while using Cabri-Geometry to draw a figure.

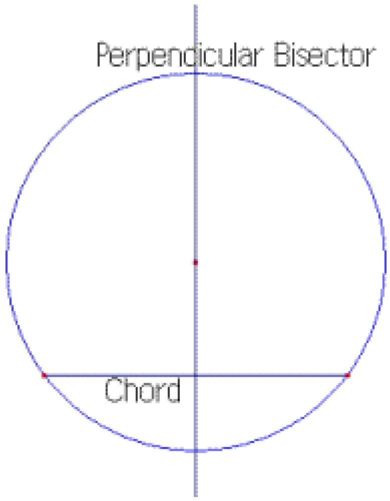

Figure7: Theorem in textbook

Conversation of Student K and Teacher T
T: "Explain why the perpendicular bisector of a chord on a circle always passes through the center of the circle."
K work: He drew a circle, then a chord, and then selected the perpendicular bisector to bisect the chord as shown in Figure7.
K: "Please look at my work! The bisector passes exactly through the center of the circle."
T: "Does it pass through the center of the circle if the chord is in a different place?"
K work: He moved the chord to various positions and showed the teacher that the perpendicular bisector passed through the center every time.
T: "How does the size of the circle affect the perpendicular bisector?"
K work: He made a new circle with a chord and perpendicular bisector. Then he changed the size of the circle.
K: "That is good. I can now draw an isosceles triangle and see the relationship between an isosceles triangle and a circle."
T: "Well done."
The student had been unable to write the proof in the notebook, but he could explain the proof by drawing the figure shown in Figure7. This conversation shows the effect of using Cabri to help understand and easily prove the theorem.We should expect that he could make his own explanation using Cabri because he had experienced the above mentioned basic construction often using Cabri one year earlier.

5.2 High school entrance examination
The following problem is from the entrance examination for Ibaraki public high schools in 1996.

Problem:
As shown in the example below of triangle ABC and it's circumscribed circle, draw a line, AF, from A that is perpendicular to chord BC. The intersection point of line BC with lineAF is point D. Draw a line perpendicular to AC from point B. The intersection point of this line and AF is point H.
Prove that triangle BFH is an isosceles triangle.

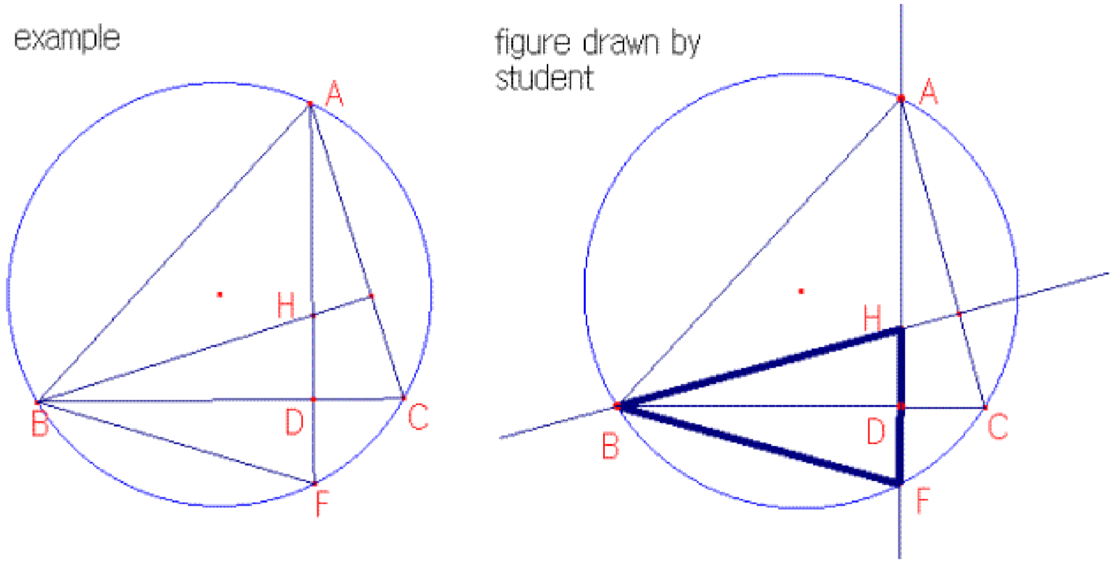

Figure 8: Entrance examination

I investigated how many students were able to solve this problem completely. I asked 40 students who took the entrance examination, and only 2 students were able to solve this problem; others lost motivation quickly, and could not understand even the meaning of problem.
The next year, I prepared the same problem for 9th grade students, allowing them to use Cabri. Many students who tried this problem were able to solve it.
To my surprise, all the students were able to distinguishing the assumption and conclusion. This difference shows how Cabri helps students understand the meaning of the problems.

6 Discussion for the Effects of Cabri for Geometry Classroom

As for the results, we could say the following: Students explored the basic construction using their own knowledge, but in different ways than those in the text book. We could not observe such kinds of reasoning in the classroom before Cabri. Students were able to explore and understand the proposition before doing the proof using Cabri. It was previously believed that the proof is the only way to understand the proposition in the geometry classroom. But it shows that the proof is not the only way, and exploring figures with Cabri also helps the reasoning up to the proof.
Laborde[3] says that three tasks promote the link between visual evidence and geometrical facts: (i)moving from a verbal description of a geometrical figure to drawing, (ii)explaining the behavior of drawing by means of geometry, which corresponds to moving from drawings to verbal description,(iii)reproducing a drawing or transforming a drawing by using geometry. He assumes that these three kinds of tasks change using Cabri.
In relation to these points, I want to discuss the following three points that helps students to develop their geometrical reasoning.

6.1 Visualizing the characters of figures clearly

When we teach construction, we usually request students to only draw the part of a circle that

is necessary for construction, because we always want the drawn figure to look beautiful. Thus, the students don't have the experience of drawing the full circle for construction. However, students need to draw a full circle to understand the characters of figures completely. Figure1-(2) shows how an isosceles triangle relates to a circle. It is assumed that 8th grade students are learning the relationship between a circle and chord, which they will use in the 9th grade. However, if a teacher gives a pre-drawn isosceles triangle to the students in 8th grade, the students do not even consider the existence of a circle.
How does the figure of a rectangle relate to that of circle?
Students can see in Figure5-(2) clearly that the diagonal lines crossing the circle must be of equal length and cross at the midpoint of the circle.

6.2 Understanding the meaning of theorem better

We ask students to prove the theorem given in the textbook. Most students are not able to write the proofs in their notebook, because they don't understand the meaning of the problem. And we are causing more and more students to dislike mathematics every year.
How can we create a situation where students draw the figure themselves, then? From the example of the aforementioned conversation, the student K could explain the theorem logically. He was a student who usually couldn't write proofs in his notebook. When students learn theorems, they feel that drawing figures is necessary. However, because of time constrains and the inability of students to draw their own figures, we give pre-drawn figures which they have to modify.
There are many students who feel frustrated and can't understand theories based on a premise. These difficulties could be overcome by teaching them how to draw their own figures, and learning the processes behind the theorem. For this, Cabri is an effective method.

6.3 Making clear what should be proving

When Students tried to solve the problem in the high school entrance examination, they were able to draw figures easily because they had used Cabri for 10 hours each year for the last two years. Most of the students drew the figures and colored them to make solving the problems easier. All students could see the differences between the assumption and the conclusion and wrote the answers clearly. There were a few students who were unable to draw figures as requested at first, but gradually increased their understanding, and drawing ability, and enjoyed drawing the figures
The most difficult thing about solving proof-problems is that students can't understand the difference between an assumption and a conclusion. There are many cases when students use a pre-given conclusion in the body of the proof instead of arriving at the conclusion from the assumptions. By drawing the figures themselves, they can understand the meaning of an assumption and a conclusion clearly. Also, the teacher can see the students' thinking processes by observing the figures they have drawn. Drawing figures helps students understand geometrical problems thoroughly.

Reference

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[3] C.Laborde *Visual phenomena in the teaching/learning of geometry in a computer-based environment. Perspectives on the Teaching of Geometry for the 21st Century*. An ICMI Study,1998

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