# A Logical Economist's Argument Against Hyperbolic Discounting

by

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**Abstract**

Although there is a large and growing literature in psychology and economics that uses experimental evidence to support the hyperbolic discounting hypothesis, this note is the first to address the question "Can we detect hyperbolic discounting by offering choices between various monetary amounts at various dates in the future to people *who also participate in markets*?" If the answer is "yes," I show that the marketplace will exploit hyperbolic discounters to the point where their wealth is zero. If the answer is "no", then the interpretations of various monetary experimental results by psychologists and some economists are illogical.

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Hyperbolic and exponential discounting are defined in Section I. Section II outlines a typical experiment that can be found in the psychology and economics literatures and is interpreted by several economists as evidence for hyperbolic discounting. I distinguish between two types of hyperbolic discounters in Section III - wealth maximizers and nonwealth maximizers and show that the former should respond to the monetary experiments in exactly the same way as a the "exponential" discounters that populate the economics literature. Section IV shows that the nonwealth maximizing hyperbolic discounters are exploited in the marketplace.

## I. Working Definitions of Hyperbolic and Exponential Discount Preferences

I adopt the working definitions of Laibson (1994). Consider the following preferences of a date t consumer for deterministic consumption sequences beginning at date t and continuing into the (potentially infinite) future:

$$u(c_t) + \beta \sum_{s=1}^{T-t} \delta^s u(c_{t+s}) \quad , u' > 0, u'' < 0, \delta < 1$$

For the hyperbolic discounter, $\beta < 1$ while $\beta = 1$ for the exponential discounter. The exponential discounter, who populates a huge number of economic models including Samuelson (1937), Barro (1974), and Lucas (1990), has “time consistent preferences” in the sense that his marginal rates of substitution do not change with the passage of time. Consider, for example, the MRS between date t+1 consumption and date t+2

consumption. For the exponential discounter, this MRS is the same from the point of view of date t and from the point of view of date t+1:

$$MRS \mid t = \frac{\delta^2 u'(c_{t+2})}{\delta u'(c_{t+1})} = \frac{\delta u'(c_{t+2})}{u'(c_{t+1})} = MRS \mid t+1$$

The hyperbolic discounter's MRS changes with the passage of time:

$$MRS \mid t = \frac{\beta \delta^2 u'(c_{t+2})}{\beta \delta u'(c_{t+1})} > \frac{\beta \delta u'(c_{t+2})}{u'(c_{t+1})} = MRS \mid t+1$$

Because their MRSs change with the passage of time, hyperbolic discounters can be said to exhibit *preference reversals*. As of date t, date t+1 consumption is not very urgent for them (relative to date t+2 consumption), but it is when date t+1 arrives.

There are other formulations of "exponential" and "hyperbolic" discounting[1], but the presence or lack of "preference reversals" is the important distinction in any formulation.[2]

## II. A Typical Monetary Experiment

Consider a typical "time preference" experiment from the psychology literature:

Question 1 Would you like $10 today or $15 tomorrow?
Question 2 Would you like $10 in 100 days or $14 in 101 days?

Actual money corresponding to the amounts above are often paid to the respondents to the questions. Examples of such experiments that have been cited in the economics literature include Kurz et al (1973), Thaler (1981), Kirby and Herrnstein (1995),

[1]For other formulations of hyperbolic discounting, see Strotz (1955), Pollak (1968), Peleg and Yaari (1973), Thaler and Shefrin (1981), and Simon (1990).

[2]For example, it is common in economic modeling to allow discount rates to vary over time, but in a way that does not lead to preference reversals. For time consistent variable discount rate models, see Koopmans (1960), Uzawa (1968), or Becker and Mulligan (1994).

Respondents often take the $10 in Question 1 and the $15 in Question 2 and it is usually assumed (and reasonably in my opinion - but the reasonableness of this idea doesn't matter for my argument) that they would take the $10 if Question 1 were reasked in 100 days.

The question posed by this note is "Does this experiment say anything about preferences?"

## III. Do Hyperbolic Discounters Maximize Wealth?

At this point, I pose a simple question to proponents of hyperbolic discounting - "Do your hyperbolic discounters maximize wealth?" I only require that proponents answer *either* yes or no to this question, but *not* "maybe."

If the answer is "yes," and hyperbolic discounters can trade in a market, then they will answer Questions 1 and 2 in order to maximize wealth. If the interest rate (inclusive of all relevant transactions costs) is greater than 50%, the wealth maximizer chooses the $10 today because $10 today can, with interest, produce more than $15 tomorrow. If the interest rate is less than 50%, the wealth maximizer chooses the $15 tomorrow.

Note that the "exponential" discounting agents that populate so many economic models are also wealth maximizers so they answer Question 1 and Question 2 *in the same way as the wealth maximizing hyperbolic discounter*; Questions 1 and 2 do not allow us to detect a hyperbolic discounter if he is wealth maximizing.

## IV. Nonwealth maximizing hyperbolic discounters?

If hyperbolic discounters that participate in markets do not maximize wealth, then let's suppose (as the psychology literature does) that hyperbolic discounters consume cash flows when they receive them rather than trading them away as the wealth maximizers might.

If some market participants are selfish and do not discount hyperbolically, nonwealth maximizing hyperbolic discounters that participate in markets will loose all their wealth. The preference reversals of these hyperbolic discounters leave them open to "Dutch books." To see how to create your own money-making Dutch book, offer the following sequence of deals to a hyperbolic discounter who answered Questions 1 and 2 as I suggested.

Deal 1 — Ask the hyperbolic discounter to promise to pay you $10 in 100 days in exchange for $14 in 101 days. He will do so because he does not

maximize wealth and, according to his answer to Question 2, prefers $14 in 101 days.

Deal 2 After 100 days have passed, collect your $10. Offer the hyperbolic discounter $10 today (the 100th day) in exchange for $15 tomorrow (the 101st day). He will take the deal because he does not maximize wealth and, according to his answer to Question 1, prefers $10 now. Pay the $10. When the 101st day arrives, collect the $15 promised in Deal 2 and pay the $14 promised in Deal 1.

You'll make $1 every time! Notice that no promises made between you and the hyperbolic discounter were broken.

There is no limit to the number or size of the Dutch books in which the nonwealth maximizing hyperbolic discounter will participate. Moreover, the hyperbolic discounter does not maximize wealth so he does not "shop around" for a cheaper "Dutch booker". In other words, one Dutch booker does not have to compete with another Dutch booker for the attention of a hyperbolic discounter because the hyperbolic discounter does not maximize wealth. Exchange between wealth maximizers and nonwealth maximizing hyperbolic discounters can only lead to the exploitation of the latter.

## V. Summary

I make two well-known points in this note. First, choices between fungible alternatives do not tell us anything about the preferences of a wealth maximizer. Wealth maximizers take whatever endowment gives them the most wealth and trade away from that endowment in the marketplace to achieve their desired consumption. Only their actual consumption tells us about their preferences. Second, nonwealth maximizers who exhibit preference reversals will be exploited in a marketplace that contains some wealth maximizers.

I use these two well-known results to show that it is illogical to use experimental evidence from psychological literature to support the hyperbolic discounting hypothesis unless one is willing to accept one of two propositions:

(a) hyperbolic discounters do not trade in markets

*or*

(b) hyperbolic discounters have no wealth

Neither of these propositions is compatible with an interesting economic analysis, so it is illogical to use the experimental results to defend hyperbolic discounting in economic analysis.

It is interesting that this logical inconsistency appears in several economics papers. Thaler and Shefrin (1981), for example, cite the experimental evidence of Kurz et al (1973) and obviously interpret the choices as non-wealth-maximizing yet don't discuss the Dutch book problem. The citations of Ainslie and Haendel (1983), Benzion et al (1989), and Thaler (1981) by Ainslie (1992), Loewenstein and Thaler (1989), and Thaler (1981) are similar: they interpret choices as non-wealth maximizing but don't explain why their hyperbolic discounters don't lose their wealth to Dutch books. Laibson (1994) and Schelling (1984), on the other hand, write down or suggest models of wealth-maximizing hyperbolic discounters yet claim that the experimental evidence of Ainslie and Haendel (1983) and others is consistent with their model.

The wealth maximization hypothesis greatly weakens the power of the experiments of Thaler (1981), Kirby and Herrnstein (1995) and others to measure preferences, but I do not suggest that hyperbolic or other preferences can never be measured empirically. When wealth maximizers are the object of observation, preferences are more easily measured if either (a) consumption is observed or (b) budget sets or observed.

Finally, I argue against the logic of using experimental evidence to support the hyperbolic discounting hypothesis, but I do not claim that choices in the experiments - or intertemporal choices more generally - are well understood by traditional economic theory. My argument is against hyperbolic discounting, but it may also help tie the noose that hangs the traditional economic approach to intertemporal choice.

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Hyperbolic discount functions induce dynamically inconsistent preferences, implying a motive for consumers to constrain their own future choices. This paper analyzes the decisions of a hyperbolic consumer who has access to an imperfect commitment technology: an illiquid asset whose sale must be initiated one period before the sale proceeds are received.Â  Hyperbolic discount functions are characterized by a rela-tively high discount rate over short horizons and a relatively low discount rate over long horizons. This discount structure sets up a conict between todayâ€™s preferences, and the preferences that will be held in the future.Â  However, from the perspective of an economist, the abundance poses a challenge. A Equivalence of Hyperbolic Discounting and Exponential Hazard. B Alternative Discount Functions. C Determining the Interval.Â  Hyperbolic discounting from TD-learning algorithms. We propose an algorithm that approxi-mates hyperbolic discounting while building on successful Q-learning (Watkins & Dayan, 1992) tools and their associated theoretical guarantees. We show learning many Q-values, each discounting exponentially with a different discount factor Î³, can be aggregated to approximate hyperbolic (and other non-exponential) discount factors.Â  Introducing the discount factor as an argument to be queried for a set of timescales is considered in both Horde (Sutton et al., 2011) and Î³-nets (Sherstan et al., 2018). Reinke et al. Under quasi-hyperbolic discounting, the valuation of a payoff falls relatively rapidly for earlier delay periods, but then falls more slowly for longer delay periods. When the salespersons with quasi-hyperbolic discounting consider the product sale problem, they would exert less effort than their early plan, thus resulting in losses of future profit.Â  A Logical Economist"s Argument Against Hyperbolic Discounting. Article. C. Mulligan.