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Editorial

Special Issue: Mathematical Neuroscience

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Natural computation relies on the biophysical substrate of the brain and the rich repertoire of dynamical behaviour that it can support. This dynamics is manifest at a range of spatial and temporal scales covering the trafficking of membrane receptors at the level of a single synapse (underlying synaptic plasticity) up to the level of travelling waves of activity as observed in whole brain electroencephalogram (EEG) and functional magnetic resonance imaging (fMRI) data. Such phenomena are likely to be described using nonlinear systems. However, in contrast to the case for more established fields, such as that of fluid dynamics, the development of fundamental mathematical models and the appropriate framework for understanding complex neural systems is still in its infancy. This is not to say that success stories in this area do not exist. Indeed this is far from the case and one only has to mention the work of Hodgkin and Huxley on modelling the action potential, Rall on the dendritic tree, and Wilson, Cowan and Amari on cortical tissue models to highlight the important steps that have already been made. We refer the reader less familiar with these examples to the short survey in [1]. Interestingly all these models are written using the language of differential equations so familiar to the Physica D audience, and which have been thoroughly analysed with the tools for describing many other physical, chemical and biological systems including bifurcation theory, pattern formation, dynamics in non-equilibrium systems, asymptotic analysis, numerical analysis, and other mathematical methods for treating nonlinear and complex systems.This provides us with a solid set of exemplars with which to define the field of Mathematical Neuroscience, a new branch of applied mathematics that coalesces mathematics with neuroscience to provide a framework for understanding the organisational principles and emergent dynamics of complex neural systems found in nature. Some more recent examples from this field include the use of symmetric bifurcation theory in an (integro-differential) neural tissue model to show that visual hallucinations can be accounted for in terms of certain symmetry properties of the anisotropic synaptic connections in visual cortex [2] and the discovery of dissipative solitons in a generalised model with physiologically motivated slow state-dependent modulation [3].

With the increased activity in Mathematical Neuroscience, and the publication of the recent book by Ermentrout and Terman entitled “Foundations of Mathematical Neuroscience” [4] it is timely for Physica D to put forward a special issue in this area. Indeed now that this field has moved on from its early beginnings in the biophysical modelling of single cells it is a challenge to pick from the many possible topics that researchers could contribute on, such as bursting patterns [5], coupled oscillator networks [6], large-scale neural dynamics [7, 8], waves and patterns [9], delay effects in brain dynamics [10], stochastic methods in neuroscience [11], or indeed novel techniques for analysis, including Evans functions [12], heteroclinic cycling [13], geometric singular perturbation theory [14], amplitude equations [15] and information geometry [16]. To provide a focus for this special issue we have therefore chosen to cover some of the topics recently discussed at the conferences in Edinburgh on Mathematical Neuroscience in 2008 and 2009 (with the next meeting in April 2010 www.icms.org.uk/workshops/neuro2010). These were run under the auspices of the UK Mathematical Neuroscience Network (mathneuronet.org.uk) and addressed the current state of research in mathematical approaches to neuroscience, covering developmental neuroscience, synaptic integration, synaptic depression, stochastic point process models of spiking activity, canards, microcircuit modelling and mean-field analysis, synchrony in cerebellar networks, population coding, spatial correlations in strongly coupled networks, cell assemblies, electrorhythmogenesis, thalamocortical networks, models of sleep/wake cycles, dimension reduction of network models and large-scale models of the ultra-slow resting brain state.

This special issue begins with a paper by Mortimer et al. [17] on neuronal development concerning the guidance of neurite fibres by molecular concentration gradients. This is a theoretical study of how fibre tips can best detect local concentration gradients. The next paper by Nowacki et al. [18] examines conditions for the onset of plateau like oscillations in a three-dimensional somatotroph cell model through a conditional slow-fast decomposition, emphasising the role of calcium-activated potassium (BK) channels. Travelling waves in realistic dendritic morphologies with excitable hot-spots are treated in the paper by Timofeeva [19], using novel techniques from mathematical physics including a "sum-over-trips" approach. Mixed-mode oscillations are investigated by Curtu [20] in a model of two neural populations and linked to the presence of a singular Hopf bifurcation. Further use of dynamical systems techniques is made by Ahn et al. [21] in order to understand the collective neuronal network behaviour of excitatory-inhibitory networks of conductance based spiking neurons, via a discrete time model for the effects of excitation/inhibition.

A novel model for binocular rivalry based on winnerless competition is introduced by Ashwin and Lavric [22] and shown to reproduce a large number of experimental results. Moving up to the scale of neural tissue, Elvin et al. [23] show how to exploit an underlying Hamiltonian structure to explain the disappearance of spatially localised (bump) states (for working memory) in favour of travelling fronts in a model of the prefrontal cortex. Kilpatrick and Bressloff [24] pursue extensions of such neural field models to include the effects of synaptic depression and more natural forms of spike-frequency adaptation and obtain analytical results on existence and stability of waves, bumps and oscillations for Heaviside and piecewise linear nonlinearities. The final paper by Faye and Faugeras [25] treats models with space-dependent delays representing axonal communication lags and makes use of techniques from functional analysis to establish existence and uniqueness of solutions for smooth nonlinearities, as well performing a Lyapunov analysis to determine asymptotic stability.

From the discussion in each of these original papers it is clear that there are many future challenges in the field of Mathematical Neuroscience, especially those that relate to space, noise, delays, feedback and plasticity in shaping the dynamic states of biological neural networks. These are very much in tune with the broader remit of Physica D and we hope that this collection of articles will spark the readers interest in working in this new field of applied mathematics.

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