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Are evolutionary games another way of thinking about game theory?

Some historical considerations

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Introduction

Evolutionary games provide *prima facie* an example of a double exchange between economics and biology, with mathematics in the background, where biology plays the role of a game theory mirror *vis-à-vis* economics.

One can picture the story in three steps: First step, several biologists pick up some concepts from the tool-box of game theory mainly used by economists, in order to model evolutionary phenomena in their fields, such as animal contests, sexual allocation among animal populations and plant growth and development (Lewontin, 1961; Hamilton, 1964, 1967). Second step, these biologists modify the definition and the content of such concepts for their own purposes. The notion of an evolutionary-stable strategy (E.S.S.), and the reference to a dynamic replicator emerged from this work (Maynard Smith and Price, 1973; Maynard Smith, 1974, 1982). Third step, the biologists' inventions are, in their turn, captured by the economists to renew and to enlarge their understanding of some basic ideas in game theory (Gilboa and Matsui, 1991; Boyland, 1992; Young, 1993). The stability of a Nash equilibrium, as well as the irrationality of a weakly dominated strategy, are revisited (Van Damme, 1991; Weibull, 1995, Samuelson, 1997). All this material gives new insights into economic competition modelling.

Evolutionary games really appeared in the field of economics at the beginning of the nineties. More than sixty years previously, Volterra published several papers on population dynamics which captured species evolution in a game-theoretical fashion (Volterra, 1926, 1927, 1931). The relationship between economics and evolutionary games is not direct. However, there was a long delay between Volterra's works on the mathematical approach to biological fluctuations and the beginning of a game theory of evolution. In addition, and strangely enough, Maynard Smith never seldom Volterra in his historical survey of this new domain.

The question does not only concern the application of game theory to such and such scientific field, but game theory in itself. Volterra developed his biological researches on animal fluctuations at the time Borel published his three key-notes on the mathematical theory of games (Borel, 1921, 1924, 1927). Volterra's reference book *Leçons sur la théorie mathématique de la lutte pour la vie* (1931) is no more than a revised and extended version of lectures given at the Poincaré Institute during the Winter 1928–1929, when he was invited by Borel. But there is no evidence that both mathematicians were aware of the similarities between the two-person zero-sum games modelled by Borel in the context of parlor games and the population dynamics where gains and losses are balanced, as in Volterra's studies. More surprisingly, this connection is still missing in many books on the history of game theory (Weintraub, 1992; Dimand and Dimand, 1996).

Moreover, the recent publication of Nash's PhD dissertation (1950a) shows that Nash had in mind, not one, but two interpretations of his famous equilibrium concept.[1] The first interpretation, labelled "Mass-action", refers to statistical populations. This requires an approximate stable frequency, which takes an immediate sense in biological matters. The other one is the well-known model based on strong assumptions about individual rationality. The second interpretation prevailed for more than thirty years among game theorists. Such a lack of interest in the first approach to non-cooperative games pointed out by Nash himself is also quite strange. However, from Nash's illuminating papers to the middle of the sixties, differential games were a shining branch of game theory, thanks to contributions by, among others, Isaacs, Berkovitz and Dresher. Indeed, these models of differential games were merely applied to military strategy situations, such as warfare, wars of attrition, pursuits and inspections (Isaacs, 1965). Once again, one cannot find any mention of Volterra's previous work in this area.

This paper is an attempt to elucidate this repeated blindness. Here we follow the intellectual history of the topic in the spirit of Mark Perlman's studies. The first part tries to explain the lack of relationship between Borel's initial programme on games and Volterra's mathematical approach to biological dynamics. The second part comes back to the origin of the Nash equilibrium and the absence of "Mass-action" echoes in game theorist circles. This historical inquiry presents arguments to show why evolutionary games can be considered as another view of games, to be extended to other solution concepts.

1 One game – two topics

Biologists like to recall the factual origin of Volterra's first mathematical work on the fluctuations of species. An Italian biologist by the name of d'Anconna, who participated in the Italian Oceanographic Committee, was puzzled by a strange observation. The proportion of predatory fishes for sale in the fish markets of Adriatic towns appeared to be higher during the First World War than in the years before

[1] H.W. Kuhn and Nasar, The Essential of John Nash (2002, pp. 79–81). Björnestedt and Weibull were the first to point out this interpretation of a Nash equilibrium and to derive from it an evolutionary dynamic through imitation (Björnestedt and Weibull, 1996).

and after. D'Anconna questioned his father-in-law, Vito Volterra, about the explanation of this observed phenomenon. Volterra's answer took the form of a system of differential equations between the two main kinds of species, the predator and the prey. According to mathematical laws, a biological equilibrium was highlighted, which was modified due to a reduction in fishing activities during the War period.

The mathematical version of the story is much more attractive for our purpose. Volterra's interest went beyond a specific question of the consequences of World War II on Adriatic fishes. This case study reactivated Volterra's previous research in applying mathematics to biological and social sciences (Volterra, 1906).[2] Such an interest was shared by Borel, who invited Volterra to give a group of lectures on this topic at the *Henri Poincaré* Institute in Paris during the academic year 1928–1929.

Indeed, the famous model predator/prey can be found in *Leçons sur la théorie mathématique de la lutte pour la vie* (1931), but only as a special configuration. The domain covered by Volterra's mathematical investigations is larger and richer. A detailed study of the determinant in linear systems, and more precisely of the quadratic forms of the linear equations, allowed Volterra to describe three types of models corresponding to three cases: the case of two species struggling for their food in a given environment (Ch. 1); the case of many species in a given environment (Ch. 2); the case of many species with endogenous and exogenous variations (Ch. 3). In this last model, delays in reactions and hereditary mechanisms in the physical acceptance are incorporated in the mathematical analysis. The strict model state predator/prey is discussed by Volterra as a sub-case in each chapter of his book. To sum up, the theoretical evolution of different species are directly derived here from the meeting of their individuals over time, under some simplified assumptions. Such a general schema is precisely what we call today an evolutionary game. Let there be no doubt: whatever the connotation associated to the term of "precursor", Volterra was a precursor of evolutionary games. Furthermore, Volterra's purpose was not only to describe in mathematical terms some possible biological evolutions, but, more ambitiously, to open the road to a dynamic approach to various kinds of real evolutionary phenomena in the biological and socio-economic spheres.

Let us turn to Borel's papers on games. Borel depicts a game in very general terms, as a social situation in which the outcome depends on chance and on the skill of the decision-makers involved in the situation (Borel, 1921). Indeed, many parlor games provide the best illustrations of such situations. But Borel was convinced that the mathematical approach to a game in this broad sense could be extended cautiously to various military and economic situations (Borel, 1924). According to Borel's definition, Volterra's examples of species' contests for survival could also be understood as games.

Moreover, Borel's mathematical starting point is the same as Volterra's. Both authors referred to the implications of mathematical properties in a symmetric linear system of equations. The title of Borel's first paper reveals his line of reasoning: "The theory of play and integral equations with skew symmetric kernels (Borel,

[2] This paper was published in the first issue of the French journal *La Revue du mois*, edited by Borel and his wife, the novelist Camille Marbo.

1921).[3] Borel supposed that a two-person zero-sum game is mathematically formalized by an A matrix with a skew symmetric determinant. The lines and the columns of the matrix are alternative "methods of playing" which are tantamount to pure strategies in the modern terminology of game theory. Hence, the problem to solve is to find a method of playing such that $aij \geq 0$, whatever the adversary's method of playing (Borel, 1921).

Thanks to his hypothesis of biological equivalence, Volterra used an identical framework in his model predator/prey, where the role of players' strategies is replaced by the population rates of reproduction (Volterra, 1931, pp. 38–39). According to such a formulation, the meeting of individuals belonging to various species in the relation predator/prey may be captured by a matrix with a skew symmetric determinant, where $aij \geq 0$. Volterra himself developed the mathematical formulation of his models in a note at the end of the Chapter 2 (Volterra, pp. 68–76). But Volterra's problem of populations' dynamics is different from Borel's strategic problem. It can be summarized in the following terms: knowing their biological rate of growth and under the equivalence hypothesis, what will be the trend of animal populations living in a same environment? The mathematical technique chosen to solve the problem led him to study separately the case where the number of animal species is even and odd.

Dynamic versus static

A slight change in the example studied by Volterra shows the links and the differences between Borel's and Volterra's modelling. Volterra considered the special case of three animal species, where the first is a prey for the second, the second a prey for the third, and the third a prey for none of the others (Volterra, 1931, pp. 63–68). Translated into strategic terms *à la* Borel, the strategic programme of the third species strictly dominates the two others. Suppose now that the first species is still a prey for the second and the second for the third, but that the third is a prey for the first; so a cycle relation exists among the three species. From a formal point of view, such a situation is exactly the same as the parlor game called La Mora (or the game paper, stone, scissors) extensively studied by Borel (Borel, 1924, pp. 205–209) and his successors (Von Neumann, 1928; De Possel, 1936, ...). But whilst the mathematical formulation is identical, in both cases choosing the minimax strategy for a player does not coincide with an asymptotic stable equilibrium among the three animal populations.

This point was made more than fifty years later by Maynard Smith in the context of the Nash equilibrium, without any reference to Borel or Volterra (Maynard Smith, 1984, pp. 19–20). Obviously, in this example, the Nash equilibrium is also a minimax solution and neither Borel nor Volterra approached their problems in terms of a Nash equilibrium. Nevertheless, Borel's comments on his mixed strategy solution reveals the real meaning of the discrepancy between games of strategy, on one side, and evolutionary games, on the other.

[3] This title is the English translation by L. J. Savage (1953) of the French original title "*La théorie du jeu et les équations intégrales à noyau symétrique gauche*", Paris, *Compte rendus de l'Académie des Sciences*, (1921).

Borel explained that, for the players of this game, the main risk is "to be discovered" by the other players. Indeed, if one of the two players knows in advance the strategy which will be chosen by the other, he will gain for certain. The mixed strategy 1/3 1/3 1/3 protects the two players against this risk. Borel showed in examples with slightly different numbers that the result remains the same, proving thus its robustness. But most interesting is its use by Borel for trying to generalize his solution to all kinds of symmetric two-person zero-sum games. According to his mathematical method, Borel extended his investigation to games with five different strategies available to each player, i.e., where the order of the skew symmetric determinant is an odd number.[4]

Individual knowledge or populations' memory

The foundations of Borel's reasoning go beyond this mathematical format. At the end of his 1924 paper, he pointed out the fact that the problem raised by a game has two components. A mathematical component, which, for Borel, could be solved by the probability calculus, and a "psychological" component, which concerns the knowledge of one player about the other players. This second component is not only irreducible to the first, but also more important than the first (Borel, 1924, pp. 220–221). Such a conviction was probably the background of his scepticisms about a general solution (Borel, 1927).

Anyway, according to this view of games, games such as "paper, stone, scissors" are extremely impressive. In such games, if one player knows the other player's way of playing, the logical solution is self-evident. The consequence is that the solution of the mathematical problem coincides with the solution of the psychological problem. Indeed, the pair of mixed strategies 1/3 1/3 1/3 guarantees each player against the risk that his strategic choice would be rightly forecast by his opponent and at the same time provides the logical solution of the situation, thanks to the probability calculus.

The game of animal population species raises quite different problems. Volterra assumed that the meeting of individuals that belong to predatory species with individuals that belong to prey species will guarantee an increase in the first population equivalent to the decrease in the second. Moreover, Volterra introduced in Chapter 3 of his book a functional relation between the number of individuals in a species and its rate of reproduction, connecting the trend of the population's magnitude to the meeting of their individuals (Volterra, 1931, p. 78). The concept of strategies *à la* Borel is here irrelevant because individual animals have nothing to choose. For the same reason, the knowledge of the play does not have in such a game situation the same meaning as in a strategic game. On the other side, the mathematical solution, when it exists, requires additional features to explain the process of adjustment through generations. In such a perspective, a game of three species predator and prey, in combinations of two, as in our example, not only has no mathematical

[4] Such a choice is determined by mathematical considerations. In the case in which the skew symmetric determinant is an odd number, the investigation is much more tractable than when it is an even number. See on the same point Volterra's comment (Volterra, 1931, pp. 59–62)

solution, but also is not supported by a consistent mechanism. Thus, one can easily imagine why Volterra, contrary to Borel, did not select this kind of example.

To sum up, for Borel, the study of a game leads to a dichotomy between a knowledge perspective ("psychology" in Borel's term) and a mathematical treatment ("probability calculus" in Borel's term). The first one is confined to the rules of the game. The second refers to the framing of the players regarding the play. This duality was recently rediscovered by Aumann. What he calls the "problem solution" approach in opposition to a "descriptive" approach through a belief system can be considered as a modern extension of Borel's intuition on strategic games (Aumann and Brandenburger, 1995; Aumann, 1999). Such a distinction between the game, as a set of rules, and the description of the players corresponding to the "play" of the game has no room in Volterra's framework, due to the inclusion of time as an explicit variable of his system. But the stability of the rules which support the mathematical solution is still to be explained by extra-mathematical considerations. This point is precisely the cornerstone for contemporary evolutionary game theorists. Thus, in spite of the mathematical similarity of their apparatus, the epistemic references are not the same in the two kinds of games. So, Borel and Volterra were right not to specify the topic in their respective works.

2 One equilibrium concept – two interpretations

The second round of the history starts with Nash. Nash, who probably ignored Borel's works on games, took up his challenge. In the same operation, Nash first extended the definition of a two-person zero-sum game to an n-person non-cooperative game, and then found the solution by reformulating Borel's mathematical problem in the language of topology. Thanks to this new concept of equilibrium point, Nash's approach seems to reconcile Borel's and Volterra's approaches to a game situation (see Nash's PhD dissertation, 1950).[5] Does this mean that, at a higher level of abstraction, the distinction previously underlined between strategic and evolutionary games disappears? The answer is not so simple.

Von Neumann before Nash

Historical evidence shows that Nash was not the first to identify the dual interpretation of a game solution. Before Nash, and twelve years earlier, von Neumann discovered the same connection between the mathematical solution of a linear dynamic system and of a strategic game in using a fixed point theorem. In his famous paper on a dynamic model of economic growth, he noted that the minimax problem raised by this model is closely related to another problem occurring in the theory of games. He expressed this thought in a footnote as follows:

[5] The content of Nash's PhD dissertation has been published in a paper under the same title, "Non-Cooperative Games" (Nash, 1951), with the exception of four pages devoted to "Motivation and interpretation". The reasons why these pages were not included in the paper are to be clarified.

> "The question whether our problem has a solution is oddly connected with that of a problem occurring in the theory of games and elsewhere (Math. Annals, 1928, pp. 305 and 307–311) ... It does not lead to a simple maximum or minimum problem ... but to a problem of stable point or minimum-maximum type, where the question of a possible solution is far more profound" (von Neumann, [1938] 1945–1946, p. 15)

In order to make a bridge with Volterra's models, let us replace von Neumann's goods by animal species and the rule for choosing technical processes of production by a biological selection mechanism. Indeed, according to von Neumann's assumptions, goods are supposed to be produced by other goods in a circular process, as predators and prey in Volterra. At the end, von Neumann's model of economic equilibrium can also be understood as an ideal case of stable animal population dynamics (steady state). One may wonder why von Neumann did not investigate more explicitly the actual connection between the games of strategy and such kinds of dynamics.

The difficulty met by von Neumann to extend the scope of the minimax theorem from two-person zero-sum games to n-persons provides a part of the answer. For von Neumann, the key of the link between strategic games and dynamic processes is to be found in the minimax rule, when its content is reformulated in topological terms. Following this research programme, von Neumann and Morgenstern were obliged to introduce several devices and fictitious figures, such as the famous fictitious player (Von Neumann and Morgenstern, 1944, pp. 505–508). From this point of view, coalitions, as they are defined in Theory of Games and Economic Behavior, may be considered as one of those devices for solving the difficulty. Such a hypothesis has obviously no direct counterpart in the domain of population dynamics derived from animal contests. Once again, the main obstacle was not in the mathematics itself, but rather in the way they are used.

The two faces of Janus

Nash's example enforces this conjecture. Like von Neumann, he set the problem raised by strategic games in topological terms, but he found another way that does not link the solution to the minimax hypothesis. Thanks to his new definition of an equilibrium point, as a set of "best strategic replies", Nash immediately generalized two-person zero-sum games as a special case of games always solvable. This advance in the mathematical understanding of a game generates several conceptual clarifications (Nash, 1950a). First, by employing the notion of solvability, Nash introduced a sharp distinction between an equilibrium point and the solution of a game. If the solution of a non-cooperative game is necessarily an equilibrium point, the reverse is not true. A non-cooperative game is not always solvable, due to a lack of uniqueness of equilibrium points.[6] Second, the solution can be strong, when the corresponding equilibrium strategies are also strong. But this is seldom the case.

[6] As every non-cooperative game has at least one equilibrium point, Nash referred to "sub-solutions" as those cases in which a game is not solvable, in spite of equilibrium points.

Finally, the solution of a game and, *a fortiori*, its equilibrium points have no reason to be optimal.

As an equilibrium point is a symmetric fixed point, such a property must also have a dynamic interpretation. Hence Nash's mathematical "*tour de force*" reopens the question of the relation between games of strategy and the representation of some dynamic phenomena. The "Mass-action" interpretation of equilibrium points sketched out by Nash at the end of his Ph.D. dissertation is not surprising. It is directly derived from his geometrical version of the topological problems raised by game situations.[7] Thus, for Nash, "mass-action" and what we may label "rational computation"[8] are two different frameworks for modelling equilibrium points in game situations.

In order to justify the section of his dissertation entitled "Motivation and interpretation", Nash noted in a laconic style, "We shall try to show how equilibrium points and solutions can be connected with observable phenomena" (Nash, [1950a] 2002, p. 78). But he was precise neither on the kind of observations (statistical data, experimental results, naïve observation, ...), nor on the nature of the observed phenomena (parlor game competitions, biological evolution, ...). His actual purpose was probably more ambitious, and at the same time less pedestrian. Perhaps he sought to give answers to the following question: what are the players' conditions on knowledge and information necessary for reaching equilibrium points which could be the solution, or at least a sub-solution, of a non-cooperative game?

In order to sketch out his answers, Nash describes two different contexts, which can be summarized as follows:

a) "Mass-action"

1a) individual players belong to a large number of possible participants (a statistical population) of the game; 2a) each participant chooses a pure strategy when he is in a playing position. Each type of pure strategy is considered as a population of players, from which an individual player is selected at random, so that 3a) the average behavior in each population defined in these terms corresponds to the population frequencies over pure strategies ("mixed strategy").

b) Rational computation

1b) players are individual persons; 2b) the game is played once and for all; 3b) the game has precise rules, which are perfectly known by each player.

In the context of "Mass-action", the relevant question becomes: Assuming 1a), 2a), 3a), what must individual players know to tend to a stable steady state of the system corresponding to an equilibrium point? As for the "rational computation", the relevant question is to be rephrased as follows: Assuming 1b), 2b), 3b), what

[7] At that time, Nash explored in pure mathematics several connections between differential geometry and real algebraic variety, i.e., "Real algebraic manifolds". *Annals of Mathematics* 86 (1952)

[8] The term "computation" is used here in a broad sense as "calculation at large" and not in its technical acceptance.

must individuals know to find the solution of the game (or, at least, a heuristic process to reach a sub-solution)?

We can see at a glance that "mass-action" and "rational computation" refer to very different contexts. Indeed, biological phenomena and animal contests are easier to frame in the mass-action format, while parlor games take their place in the "rational computation" context. But the lack of a transmission mechanism does not allow a direct use of "mass-action" to model any specific evolutionary phenomenon (biological, as well as social). As for the parlor games, if their main features are well captured by the stylized rational computation context, this is not sufficient to guarantee the relevance of Nash's questioning. The knowledge problem of the players raised, for example, by the chess game, cannot be reduced to the rational computation of a solution/or sub-solution *à la* Nash.

Let us compare the main entities of a game in these two contextual situations. Players do not have the same definition in the "mass-action" and in the "rational computation" scenarios (see 1a and 1b). A strategy in both contexts does not mean the same thing. Whereas its definition is unique in the "rational computation" context, it changes in the "mass-action" context according to its "pure" or "mixed" acceptance. Whereas the individuals implement the same pure strategy during their play, the mixed strategy which results from their choice can fluctuate until the end of the game. Such an ambiguity has recently been criticized and discussed by Rubinstein from another point of view (Rubinstein, 1991). Finally, the solution (or the sub-solution) of a game belongs to the rules of the game in the "rational computation" context, but not necessarily in the "mass-action" context, for which "solutions have no great significance" (Nash, [1950] 2002, p. 78). A different question investigated by means of models that refer to concepts differently defined leads necessarily to different answers. Therefore, it is not very surprising that the cognitive conditions required to reach an equilibrium point in a game pictured in a mass-action model are very different from the cognitive conditions required to attain the solution of a game described by a rational computation model.

Nash conjectured that, in a mass-action model, individual players only use empirical information from the moves actually played for choosing the best strategy among the pure strategies at their disposal. They are not aware of the rules of the game and of its structure. As opposed to such models, individual players in "rational computation" models must have a complete knowledge of the rules of the game and the epistemic ability to make mutual rational expectations, which are also supposed to be rational. But they do not need, in principle, to take into account empirical information provided by the play of the game.

Nash's original conjecture was probably right. The different belief systems associated with a Nash equilibrium in a "rational computation" situation are based on knowledge conditions of rationality. The learning and/or imitating processes coupled with a Nash evolutionary equilibrium are mainly derived from inductive (or adductive) procedures. Furthermore, Nash suggested that the line of demarcation between the two interpretations is not to be found in the number of individual players, but rather in the time dimension. Indeed, he noted that, in the "mass-action" interpretation, "the populations need not be large if the assumptions still hold" (Nash, [1950a] 2002, p. 80). In other words, in a game with few individual players who do not know well the rules and the structure of the game, rational computations can be replaced by inductive information if the game is played over a sufficiently long period. Thus a bridge was suggested by Nash as between repeated experimental games and evolutionary games as soon as experiences are accumulated.

By comparison with Borel and Volterra, Nash's suggestions mark a significant step in the understanding of the link between strategic games and dynamic phenomena. However, Nash did not refute Borel's and Volterra's implicit positions. For Nash, a "mass-action" and what we call a "rational computation" are two classes of different models, while for Borel and Volterra, the mathematics of animal fluctuations and the mathematics of parlor games were two separate topics. But Nash added the notion that "mass-action" and "rational computation" were both derived from one and the same mathematical notion, the equilibrium point. Thus, Nash shifted the sense of the initial question. No problem, an equilibrium point can be relevant in the two types of models. But does this also mean that a Nash equilibrium is the unique solution concept for all kinds of games?

Solution and solutions

Nash was aware of the challenge. First, the distinction between "solution" and "sub-solution" underlines the fact that the identification of an equilibrium point is not sufficient for solving every non-cooperative game, because a game frequently has many equilibrium points. Concerning this difficulty, Nash made the following suggestion:

> "In an unsolvable game it sometimes happens that good heuristic reasons can be found for narrowing down the set of equilibrium points to those in a single sub-solution, which then plays the role of a solution" (Nash, [1950a] 2001. p. 23).

Which kind of heuristic procedure did Nash have in mind at the time? Does it lead to a meta-criterion along the lines of Harsanyi and Selten (1988),[9] or to a contextual empirical information, in the spirit of Shelling's focal points? Nash never came back to the question. Nevertheless, the problem raised by the multiplicity of Nash equilibriums remains unsolved until now.

Second, Nash extended equilibrium points as a solution concept to cooperative games by means of his models of non-cooperative bargaining in two seminal papers (Nash, 1950b, 1953). This is a well-known story which is summarized today under the title of the "Nash programme" (Binmore and Das Gupta, 1987). Thanks to this approach, the cooperative solution of a game can be analyzed as the result of a non-cooperative game of negotiation . All these developments may be understood in terms of "rational computation" models; Nash never used the "mass-action" canvas to enlarge the domain of the equilibrium solution concept. A more detailed investigation of mass-action's main features can probably explain such an interesting thing.

[9] At the end of his life, Harsanyi radically changed and moved to a completely different theory of equilibrium selection which he never had the opportunity to express (i.e., Selten's postscript to the Special Issue: John C. Harsanyi memorial, *Games and Economic Behaviour* Vol. 36: 1 (2001).

According to Nash's simplified assumptions, the solution of the dynamic situations described in the "mass-action" framework is a stationary distribution of populations. Distributions other than Nash equilibriums can also be stationary, as was observed by Bjornerstedt and Weibull (1996). On another hand, if we require stronger conditions of stability for a solution, as for instance an asymptotic stability, it is well-known that the Nash equilibrium concept is no longer sufficient, because many Nash equilibria are not asymptotically stable. So, whereas an equilibrium point must always have a "mass-action" interpretation, the coincidence between an equilibrium and a solution of such games remains rather problematic.

The situation is quite different in the context of a "rational computation" model. In a strictly non-cooperative game played once and for all, a weaker condition than "best replies" seems difficult to imagine for a strategic solution. Almost all concepts proposed as alternatives to Nash equilibria are refinements for narrowing the set of equilibriums. But the most interesting point is that the model of non-cooperative bargaining is necessarily supported by strong assumptions on players' knowledge. In its interpretative version, each player is supposed to be fully informed of the structure of the game and the pay-off functions of the other players. In addition, each player must have sufficient information in choosing optimal threats and demands. Such assumptions are translated into axiomatic conditions in the axiomatic version of the bargaining system (Nash, 1953). Anyway, in Nash's two-person negotiation model, deterrence threats imply, at least, three levels of rationality, namely, 1) the players are rational; 2) each player knows that the other is rational; 3) each player knows that the other player knows that he is rational.[10] Hence, the deterrence mechanism is based on continuing conditions of rationality, which cannot easily be directly translated in terms of mass-action.

There is another reason why Nash and his successors addressed the question of equilibrium in terms of "rational computation", rather than in terms of "mass-action". The notion of a solution for a non-cooperative game is precise in the first case, but vague in the second (Nash, [1950a] 2001, p. 78). It appeared in the discussion that "rationality" was really the cornerstone of the question. Good examples are provided by several Nash equilibrium refinements, which were introduced by many game theorists after the seventies. The concept of "perfect equilibrium" proposed by Selten during this period to challenge Nash's equilibrium concept is perhaps the best illustration of the dominant way of reasoning (Selten, 1975, 1978). In order to avoid apparent inconsistencies resulting from Nash equilibrium applications as a game solution, Selten proposed to reinforce its rational conditions. Roughly speaking, a "perfect" equilibrium eliminates all kinds of dominated moves in strategies which are supposed to be "irrational".[11] Such an exercise has obviously no meaning in a mass-action universe.

[10] Such cognitive conditions on rationality are stronger than for a simple Nash equilibrium which does not require even a mutual knowledge of rationality, at least in a two-person game (Aumann, 1992; Aumann and Branderburger, 1995).

[11] It must be noted that Selten is still not convinced today of the relevance of his "perfect equilibrium" for solving games, because he moves to inductive solutions which do not belong to the rational computation modelization.

When Nash published his major papers, the equilibrium point did not focus the research on game theory. However, a group of game theorists applied differential calculus *à la* Borel and Volterra to the new format of von Neumann and Morgenstern's theory of games. This group worked through the Rand Corporation and various other military think tanks. The dynamic dimension was explicitly taken into account in their models. Moreover, Nash was more or less connected with several members of this group, such as Dresher when he worked at Rand. Therefore, one can reasonably wonder why such differential games were never connected to the mass-action approach.

The answer is to be derived from the purpose of these groups. Differential equations are applied here for practical reasons, in order to find operational solutions to military problems, combat, pursuit, escalation and, more generally, warfare situations. Isaacs has well summarized this diagnosis. Topology and formal logic do not provide techniques for obtaining practical answers to these questions. Even in the simple case of two-person games, the matrix quickly becomes too large to be tractable. Therefore, it is convenient to treat such situations as games in which two adversaries are confronted with sequences of continuous decisions in a calculable pattern, thanks to differential analysis (Isaacs, 1965). Differential equations in this respect are no more than a practical procedure. Nash also used this tool for computational purposes in a Rand Corporation Memorandum (Nash, 1954). When Isaacs speaks of a solution, he does not refer to a theoretical concept, but to an operational answer. Indeed, those researches on differential games supported by the Rand Corporation and the Center for Naval Analyses develop dynamic studies, but without any evolutionary connotation. Differential games are one thing, evolutionary games another.

Conclusion

Some interesting ideas emerge from this historical survey. Borel, Nash and Volterra studied consciously or unconsciously non-cooperative games. Thanks to Nash, we know that any non-cooperative game can be considered from two points of view. First, commonly linked with a strategic approach, the issue of the game depends on players' rationality. In a more sophisticated way, we can speak today of the logical understanding of the players via an epistemic logic. Second, frequently attached to an evolutionary perspective, the issue of the game is determined by the dynamic stability of its trajectory. A Nash equilibrium is defined as a rational situation (a set of best strategic replies), by reference to a stable situation. In a Nash equilibrium, no player has an incentive to deviate. Rationality and stability are in this case intimately but ambiguously connected. However, more reflection reveals that in a Nash equilibrium, rationality is finally used in a relatively weak acceptance (Schmidt, 2001) and that the stability condition is not always sufficient to be applied to dynamics.

Such observations suggest a further question: Beyond Nash equilibrium, does a more general relation exist between the rational cognition of the players and the stability of the game at issue? More precisely, what kind of properties relate these two poles of a game: substitution, complementarity, ...? A lot of material is now available allowing for a more systematic investigation.

Following this line of reasoning, it seems established now that the "Nash programme", in spite of its strong attraction, does not suppress all other solution concepts for cooperative games. Indeed, a cooperative game cannot be considered as strategic and nothing is said about its dynamics. But their solutions require rationality conditions and some of them, though not all (see the Shapley value), refer also to stability conditions. Unfortunately, the cognitive dimension of this rationality has still not been adequately investigated and their evolutionary trend has not been studied in a dynamic perspective until now. It could be a starting point for a fruitful research programme.

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The evolu-tionary game theory way is to look at the environment, see how the various strategies fare and how the environment is therefore likely to change when we take evolutionary pressure into account. If we start at a point in time whereby more people drive the left than the right, then those agents who adopt this strategy are doing better than those who stick to the right. So.Â The second and third parts explain the evolutionary game theory model in one and two population settings. 2.1 Notation and basics of non-cooperative game theory.Â In this setting we can think of the environment (population state) as describing the prevalence of each of the pure strategies in the population. Hence there is a one-to-one-correspondence between the set of population. With the development of game theory, evolutionary game theory provides a practical method to solve these problems. Renegotiation Strategy of Public-Private Partnership Projects with Asymmetric Informationâ€"An Evolutionary Game Approach. Article.Â Occupational-disease appraisal is a kind of way for laborer to maintain their rights and interests. The laborer applies and the employing units provide materials, and then the relevant departments give the identification based on the materials that the laborer and the employing units provided. If the laborer is really suffering occupational disease, the employing units have to pay the laborer the corresponding compensation, and the laborer's rights and interests can be successfully maintained. Evolutionary game theory is a way of thinking about evolution at the phenotypic level when the fitnesses of particular phenotypes - John Maynard Smith quotes at AZquotes.com.Â â€œEvolution and the Theory of Gamesâ€, p.1, Cambridge University Press. â† Prev John Maynard Smith Quotes Next â†'. facebook. twitter. googleplus. email. linkedin.Â There is a mistake in the text of this quote. The quote belongs to another author. Other error. Comments: Email for contact (not necessary): Authors. Topics. Picture Quotes.