A Collaborative Model for Calculus Reform—A Preliminary Report

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Abstract

For the past two decades, both pros and cons of calculus reform have been discussed. A question often asked is, "Has the calculus reform project improved students' understanding of mathematics?" The advocates of the reform movement claim that reform-based calculus may help students gain an intuitive understanding of mathematical propositions and have a better grasp of the real-world applications. Nonetheless, many still question its effect and argue that calculus reform purges calculus of its mathematical rigor and poorly prepares students for advanced mathematical training. East Asian students often rank in the top 10 of TIMSS and PISA. However, out-performing others in an international comparison may not guarantee their success in the learning of calculus. Taiwanese college students usually have a high failure rate in calculus. The National Science Council of Taiwan therefore initiated several projects in 2008 for improving students' learning in calculus. This paper provides a preliminary report on one of the projects, PLEASE, and discusses how it was planned to respond to the tenets of calculus reform movement.

Introduction

It has been over two decades since the Tulane Conference, the birthplace of calculus reform, was held in 1986. The appeal made by the conference—Toward a Lean and Lively Calculus (Douglas, 1986)—not only initiated the calculus reform movement in the United States, but for the first time ever, motivated numerous research mathematicians to engage in curriculum development. Despite several promising empirical findings having been reported, a widely held conclusion on the effect of calculus reform is still in debate. The equivocal consequence is due to the universal goal of teaching calculus being unattainable and a standard evaluation method is lacking. Calculus curriculum came under scrutiny for several reasons. First, traditional training in calculus, stressing rote calculating and practicing, hinders students from gaining a higher level of conceptual understanding and fails them in studying advanced mathematics course. Second, somewhat related to the first reason, a high failure rate in calculus forces college students to leave engineering or keeps them from choosing a mathematics-related career. Third, the faculty outside mathematics usually complains that students are ill-prepared to apply learned skills and concepts to solve practical problems. Reform effort in calculus curriculum aims to restructure content and develop tools to fix aforementioned pessimistic situations. We will make a brief review of calculus reform projects, followed by a preliminary report on the PLEASE project supported by the National Science Council of Taiwan.

A Brief Review of Calculus Reform

Rooted in its rigorous development in history, traditional instruction in calculus is conducted in logical order in which proving theorems and propositions deductively, based upon definitions and lemmas, plays a critical role; and working exercises with paper and pencil become the dominating mode of learning. This formal approach secures the foundation of calculus, but at the expense of students' intuitive understanding of the discipline. Some mathematicians thus made an urgent call for restructuring calculus curriculum.

Responding to the appeal of the Tulane Conference, Brown, Porta, and Uhl (1990a) reduced calculus curriculum by deleting several topics, such as Roll's Theorem and Riemann sum definition of integral, and integrated technology into the curriculum instead. They claimed that certain topics contained in traditional textbooks are only to fool students into the belief that they have learned something (Brown, Porta, & Uhl, 1990b). Among all reform curricula,

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Harvard Calculus Consortium (Hughes-Hallett & Gleason *et. al*., 1992, 1994) is the most widely adopted text and has received the greatest attention. Harvard Calculus Consortium reflected the reformed idea of "The Rule of Three" declaring that every topic should be presented in geometrical, numerical, and algebraic ways. Furthermore, it de-emphasized deductive symbolic reasoning by decreasing some sections, and stressed students' ability of application by connecting formal definitions and procedures with practical problems. For instance, *Calculus*: *Early Transcendentals* (Stewart, 1999), one of the best-selling textbooks and regarded as the most traditional textbook at that time, used five sections for discussing the concept of limit, whereas there was only one section on limits in the Harvard Calculus Consortium.

Johnson (1995) reported on the effect of the Harvard Calculus Consortium at Oklahoma State University by comparing students' performance in reformed and traditional calculus. He indicated that reformed classes outperformed traditional classes in Calculus I and II. However, traditional Calculus I students' subsequent performance in mathematics-related courses were better than their counterparts in reformed Calculus I classes. Furthermore, 44% of reformed Calculus I students changed to traditional Calculus II programs and only 18% of traditional Calculus I students shifted to reformed Calculus II. Baxter, Majumdar, & Smith (1998) also surveyed reformed and traditional calculus students' achievement in the Math-ACT and found that traditional Calculus I students' average grade was slightly higher than that in the reformed Calculus I, but only 52% of traditional Calculus I students passed the exam, significantly lower than reformed Calculus I students' passing rate of 64%. As for succeeding performance, reformed Calculus I students surpassed the traditional students in Physics I and Calculus II, yet traditional Calculus I students did better in Physics II and Calculus III. These outcomes seemingly suggest that the effect of the reformed curriculum may decline as the difficulty of the content increases. It appears that the experimental consequences of the reformed curriculum are hard to summarize in a single sentence. Silverberg (1999) proposed that the reformed curriculum may be more effective for those with a weak mathematics background.

Critics and Influence of Calculus Reform

Despite its success in several aspects of helping students to master calculus concepts, some professional mathematicians remain doubtful as to what has really been achieved by reform movements. While responding to Mumford's (1997) arguments, Klein and Rosen (1997) condemned reform supporters by saying that they create a straw man—the traditional calculus curriculum—and blame all faults on it. Reformers put forth various solutions, such as eliminating theories and increasing use of computers, without any scientific evidence. What if they were wrong in identifying the cause of students' failure? In their eyes, traditional calculus actually gives students the opportunity to have a deeper understanding of the subject and reform texts hinder motivated students from developing advanced thinking. Klein and Rosen satirized that calculus reform movements are not for the millions but $millions.

Feffer and Petechuk (2002) took a neutral stance. They agreed that reform curriculum may help students be more capable of connecting mathematics with the real world but, in their eyes, democratizing the curriculum by reducing its rigor actually is just watering it down. Feffer and Petechuk emphasized that calculus should not be expected to apologize for being difficult because skills will help students succeed.

Though calculus reform receives praise as well as criticism, there are some signs revealing its positive influence on textbook development. *Calculus*: *Early Transcendentals* (Stewart, 1999) was once regarded as the representative of the traditional textbook, yet Stewart (2006) claimed in his recent edition that:

> When the first edition of this book appeared eight years ago, a heated debate about calculus reform was taking place. Such issues as the use of technology, the relevance of rigour, and the role of discovery versus that of drill, were causing deep splits in mathematics departments….In this third edition I continue to follow that path by emphasizing conceptual understanding through visual, numerical, and algebraic approaches.

> The principal way in which this book differs from my more traditional calculus textbooks is that it is more streamlined….I don't prove as many theorems….(Stewart, 2006, xiii-xiv).

It appears that, as Stewart (2006) pointed out, both reformers and traditionalists have realized that enabling students to understand and appreciate calculus is their common goal. We are convinced that any reform effort should keep track of this common goal.

PLEASE Project

The PLEASE project adopts a collaborative model consisting of five individual projects conducted by Mathematics and Engineering faculties at two technological universities in Taiwan. In addition to reform calculus curriculum itself, it establishes a system for assisting students' learning not only in pre-calculus, but also in subsequent mathematics-related courses. The title PLEASE stands for six main themes of this integrated project: (1) P—pre-calculus, (2) L—low achievers' learning, (3) E—e-learning, (4) A—assessment, (5) S—statistics and calculus, (6) E—engineering mathematics and calculus. The PLEASE project can be divided into three components: PEA, LEA, and SEA.

PEA component: Technological universities in Taiwan mostly recruit students graduating from vocational high schools, stressing more skill training and practical knowledge. Such an instrumentalist approach may restrict college freshmen's conceptual understanding of fundamental topics in calculus, such as the concept of functions and limits. By following the Rule of Three (every topic should be presented geometrically, numerically, and algebraically), the PEA (pre-calculus, e-learning, and assessment) component combines Calcai, a graphical software, and Mimic Builder, an e-learning device, to develop an e-learning tool for pre-calculus. It enables non-technical users to create the e-learning courses by using the PowerPoint file and working with the assistant Tablet digital pen. The 2-layer slide design also lets the teaching process proceed more smoothly. Furthermore, a web-based test bank is established for assessing college freshmen's concept knowledge in pre-calculus and evaluating the effect of this e-learning tool.

LEA component: High failure rate in calculus is not uncommon in Taiwan's universities. Despite their outstanding performance on international assessments in mathematics, such as TIMSS and PISA, Taiwanese students' attitudes toward mathematics have been reported to be very low (Mullis, Martin, & Foy, 2008). Poor attitudes, to a great extent, weaken these college freshmen's driving force to learn calculus, which is essential for their majors. The LEA (low achievers, e-learning, and assessment) component attempts to construct an auxiliary environment, including computerized adaptive diagnosis evaluation system and teaching assistants, to enhance low achievers' learning. In order to identify difficulties in learning calculus, they are asked to respond to items from the web-based evaluation system. Because the computerized adaptive test is knowledge-structured and hierarchical, we may locate their obstacles in learning calculus and develop a tailored curriculum. Moreover, selected teaching assistants are trained to execute the tailored curriculum and serve as instructors outside the classroom.

SEA component: One of the criticisms of calculus reform is that reformed curriculum may show deficiencies in preparing students to take advanced mathematical courses. The SEA (statistics and calculus, engineering mathematics and calculus, and assessment) component deals with this issue by restructuring calculus curriculum to help students make a connection between calculus and subsequent mathematical courses, such as statistics and engineering mathematics. Several fundamental concepts in statistics (e.g., the expected value of the function of discrete stochastic variable and continuous stochastic variable) require sophisticated understanding of infinite series and definite integral, which are difficult for business majors to figure out. A particular emphasis on these topics will be made to fit their future needs in studying statistics.

Similarly, engineering majors usually have trouble grasping complicated concepts and processes of engineering mathematics such as differential equations, Fourier series, and Laplace transform, all of which entail a solid background in integrals as well as in differentials. In our calculus curriculum, students are trained to construct and solve mathematical models of given realistic problems by introducing to them the solutions of basic types of differential equations, Fourier series, and Laplace transform.

Conclusion

The PLEASE project assumes a collaborative model not only for reforming calculus curriculum itself, but also for establishing an e-learning and assessment platform. Three main components (PEA, LEA, and SEA) cover an extended range of curriculum from pre-calculus to post-calculus courses (Figure 1).

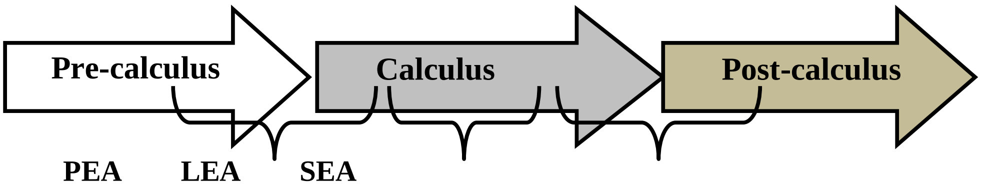

Figure 1

In order to keep away from the debates occurring in calculus reform, we set the issue of rigor aside and take students' learning in subsequent mathematical courses into account by stressing intuitive understanding and application of calculus. This approach, however, may only be conditionally applied. Students in our projects have graduated from vocational high schools and are studying at technological universities, which are usually less theoretical in their professional training. Our chief belief is any curricular reform effort is destined to fail if the curriculum itself is the only concern and an auxiliary system for supporting reform curriculum is lacking. The output of calculus reform should not be a single textbook but a holistic learning platform bridging the gap between preliminary and advanced concepts and knowledge.

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A Special Calculus Survey: Preliminary Report 215 Richard D. Anderson and Don Loftsgaarden Two Proposals for Calculus 216 Leonard Gillman. Calculus in Secondary Schools 218 NCTM-MAA Joint Letter Calculators in Standardized Testing of Mathematics 219 College Board-MA A Recommendations.Â Calculus for a New Century is itself a middle chapter in a long process of calculus reform. It builds on a much smaller workshop organized by Ronald G. Douglas at Tulane University in January, 1986 which led to the much-cited publication Toward a Lean and Lively Calculus (MAA Notes No. 6, 1986). Shortly afterwards, the Mathematical Association of America appointed a Committee on Calculus Reform chaired by Douglas to make plans for an appropriate follow-up. As healthcare reform unfolds, new business models and restructuring will emerge to manage costs while delivering compassionate, quality care. On the macro level, healthcare organizations must look far ahead to understand how the unfolding future impacts their current structures and business models. They urgently need to seek out opportunities to reduce costs.Â CCLâ€™s leadership model for healthcare transformation focuses on the development of six organizational capabilities that can help to create a collaborative leadership mindset. It is based on the ultimate goal of developing an interdependent leadership culture that will lead to quality, compassionate patient care in the face of the adaptive challenge. The process of creating a mathematical model is ac-tually nothing more than the arrow in the diagram going from pictures to symbols. Mathematical modeling is the jump you make from the visual information you have created to information contained in your formulas. Letâ€™s summarize the problem solving process. You start with a de-scription of a problem that is presented to you mainly in the form of words. Instead of trying to jump directly from words to symbols, you jump from words to pictures. Once you have a good picture, you jump from pictures to symbols.